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Markets of joint products: A theoretical model and policy implications ☆

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Abstract

The article considers the phenomenon of price behavior in markets of joint products. It shows that conclusions about the nature of economic entities' behavior on this type of market could be inaccurate if the characteristics of these markets, such as joint costs, are not taken into account. For this purpose, a theoretical model, built according to basic microeconomic principals, is applied. This model provides an opportunity to reveal — without further new institutional analysis — that the reason for price deviation from a competitive level does not always lie in actions restricting competition.

JEL classification: D4.
Keywords: joint products, joint costs, competition.

1. Introduction

This analysis is motivated by a sound antitrust case brought by the Federal Antitrust Services of Russia (hereafter FAS) against Russian companies in the chemical industry in 2011, which was closed by final court decision in April 2015.[1]

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[1] Open Joint-Stock Company (OJSC) Edinaja Torgovaja Kompanija (United Trade Company), Ltd. Sibmanagement Group, Ltd. Sibmanagement, OJSC Himprom (Volgograd), Closed Joint-Stock Company (CJSC) NPO Reagenty, OJSC Sajanskhimpast, Ltd. Sibirskaja Himicheskaja Kompanija (Siberian Chemical Company), Ltd. Usol'ehimprom, group of companies: Ltd. "Sibur", CJSC "Sibur Holding", OJSC "Sibur-Neftehim", group of companies: OJSC Novomoskovskaja Akcionernaja Kompanija Azot (New Moscow Joint-Stock Company Nitrogen), OJSC Mineral'no-Himicheskaja Kompanija EvroHim (Mineral and Chemical Company EvroHim),

(*continued on next page*)

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Three interconnected cases were brought against companies operating in the markets of chlorine, liquid caustic soda and polyvinylchloride (PVC). The case on the market of chlorine was brought on the basis of an investigation launched by the unscheduled inspection of the Caustic company (Volgograd), when documents proving a price maintenance and market sharing agreement were revealed. The investigation was continued by other unscheduled inspections when corresponding documents were found. According to these documents, the cartel on the market of chlorine took place over 3.5 years and led to an increase in prices by a factor of 3.5 times. An investigation in the market of caustic soda was also based on unscheduled inspections and economic analysis that revealed a cartel operating since 2005 throughout the entire country. Unscheduled inspections and economic analysis were also used for an investigation on the market of PVC. It was revealed that the cartel on this market was already at least five years old (Artemiev et al., 2013, pp. 35–39).

Despite the fact that in each case the antitrust authority possesses documents proving, as it seems, agreement, the conclusions of appellate courts where decisions by the FAS were contested were not unambiguous, which is the consequence of the peculiarities of these markets.

One of the key arguments of the FAS proving a price maintenance agreement is the fact that the domestic price of liquid caustic soda exceeds its export price (Artemiev et al., 2013, pp. 66–67). According to the antitrust authority, this fact implies intensive competition on the foreign market and weak competition on the domestic market.

The case on the market of caustic soda ended up with the win of companies, and one of the decisive arguments was that there is constant relationship between the production of caustic soda and the production of chlorine, and this relationship explains the behavior of companies in corresponding markets.

The main goal of this study is to show, basing on the neoclassical model, that the difference in prices pointed out by FAS can be a consequence of the peculiarities of production and is not necessarily caused by a market structure. In turn, the approach of the antitrust authority that disregards these particularities could lead to errors of law enforcement (in this case—a type 1 error—unreasonable accusation).

The key particularity in the production of caustic soda is that the latter is produced in fixed proportion with chlorine. If within a single production process two or more products are manufactured, these products are called joint products. Costs of production of this type of product are joint until the split-off point. In turn, the split-off point is the stage in the production process after which joint products follow their own patterns. There are joint products in fixed and variable proportions. The chemical industry provides a variety of examples of joint products both in fixed and variable proportions (Kreps, 1930). The production of chlorine and caustic soda is one such example.

The significance of the question under review goes beyond not only the important and particular case of caustic soda and chlorine but also beyond the chemical

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(continued) Ltd. Novomoskovskij Hlor (New Moscow Chlorine), group of companies: OJSC Bashkirskaja Himija (Chemistry of Bashkiria), OJSC Kaustik (Caustic) (Sterlitamak), group of companies: CJSC Renova Orgsintez, OJSC Himprom (Novocheboksarsk), group of companies: Ltd. Nikohim, OJSC Kaustic (Caustic) (Volgograd), group of companies: OJSC GaloPolimer, Ltd. GaloPolimer Kirovo-Chepetsk, group of companies: Ltd. Torgovyj Dom Himprom (Firm Himprom) (Kemerovo), Ltd. Production Association Himprom (Kemerovo).

industry. In addition, analysis of this question shows that the tradition of hostility in antitrust revealed by Coase more than forty years ago (Coase, 1972) has a more complicated structure because even disregarding the tools of New Institutional Economics and based on the analysis of Neoclassical Economics it is possible to explain “anomalies” that form the occasion for accusation of market participants in infringement of competition law.

The first part of the article provides a review of the main ideas of the concept of joint products based on other studies on this topic. The next three parts represent the model considered equilibrium conditions under competition and monopoly on markets of joint products, different reasons for a glut and wastage. In addition, the possibility of separate processing and transportation to a distant market (open economy) of a co-product is taken into account. The conclusion provides the results of the analysis.

2. Joint products

The question of joint products is a subject of a series of works based on microeconomic models taking into account their different particularities. In this way, Deutsch (1965, pp. 397–401), using the example of the markets of chlorine and caustic soda in Canada, shows that complicated transportation and the moderate cost of the wastage of a glut of one of the joint products influence the parameters of markets. According to the Deutsch, the pricing policy of a company manufacturing these products is reminiscent of price discrimination on a territorial basis, but in reality, it could be the consequence of high transportation costs and a problem of the wastage of a glut.

Some articles concern the problem of cost allocations to joint products. Manes and Smith (1965, pp. 31–35) suggest an approach where costs are allocated according to the marginal revenue from sale of each of them. The authors also define the difference between co-products and by-products. They offer to consider a product as a by-product if it does not contribute to the sum of marginal revenue at an equilibrium point. This product should be disposed of at marginal revenue equal to zero. This corresponds to the cost allocation such that all costs are applied to a major product and the production of a by-product until the split-off point is assumed to be free of charge. If a by-product is separately processed (in our example—production of chlorinated derivative, for instance PVC), then the volume of its sale should correspond to the level where its marginal revenue equals the marginal cost of separate processing. Another article (Jensen, 1974, pp. 465–476) that also concerns the question of cost allocations to joint products considers the case when the price of a by-product (with the same definition of a by-product) could differ from the marginal cost of separate processing. The reason is that the price of a by-product depends not only on cost of separate processing but also on opportunity cost of selling a by-product at the split-off point. If this price is negative, meaning that the alternative of separate processing is wastage involving cost, the price of the separately processed good is lower than marginal cost of separate processing.

In turn, Walters (1960) offers to employ the demand function for joint products in the form of probability distribution that enables high uncertainty level on markets for these products to be taken into account.

In one more article concerning joint products (Colberg, 1941, pp. 103–110), the author explains the appearance of a glut (the difference between output and the volume sold) not only by limited demand for a product but also by the reasoning of a monopolist maximizing profit. As far as the wastage of one of the joint products enables interdependence of their sales to be avoided, a company can decrease sales of one product keeping the sales of another product constant. In turn, Blair and Haynes (2012) extend the analysis of markets of joint products adding monopsony on one of them and showing how this transformation of a model influences wealth, market equilibrium, and the market power of a company, if it has one.

Graham and Green (1984) use the concept of joint products (products manufactured from one input[2]) in their analysis of households, where input is time and joint products are home production and leisure.

Finally, several articles consider the influence of government policy and market fluctuations on joint production (Houck, 1964; Piggott and Wohlgenant, 2002). In particular, they study the relationship between demand for joint products and aggregate demand for their common input.

Each of these models considers one or several characteristics of joint products, but does not study their interrelationship and a general effect on parameters of markets. This reveals a theoretical gap that impedes the analysis of the question under review using existing models. The new approach to modeling the decision-making process of a company manufacturing joint products and equilibrium on corresponding markets, based on the experience of preceding works, is elaborated. The new model takes into account the peculiarities of production both in fixed proportion equal to one and in other fixed proportions, joint costs, cost of wastage (including prohibitively high cost of wastage), and the possibility of separate processing and transportation of a glut. The distinctive features of the new model are the following. First, it considers different reasons for a glut that enable the comparison of outcomes on competitive markets and monopolistic markets. Second, it takes into account the possibility of separate processing and transportation as the ways to dispose of a glut on a different market that is an alternative to wastage. This enables the comprehensive analysis of the effect of a glut on the parameters of markets.

The new model reveals probable law enforcement errors that take place on markets of joint products because the peculiarities of these markets are not taken into account by the antitrust authority.

3. Equilibrium on markets of joint products

By their nature companies with joint production are multiproduct companies because they receive profits from selling several products. The profit of such a firm is composite and consists of profits from selling of each product manufactured by the company (aggregate profit). Assume that there is no relationship between joint products on the demand side, but by definition their production and hence supplies are interrelated, which means that the production of one product implies the production of another product (in particular, the amount in the case of fixed proportion of joint production). Hence, such products have joint costs and joint supply. These particularities

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[2] Strictly speaking, the general idea of the analysis does not change if there is more than one input. For example, in the case of chlorine and caustic inputs there are salt brine and electricity.

of joint production transform the maximization problem of companies producing this type of product as follows. First, such a company cannot determine the optimal level of output of each product separately. The output choice of one product that maximizes profit (a major product) implies output of other products (co-products) through fixed proportion. The optimal output of a major product should maximize a company's aggregate profit, meaning that it should take into account the consequent output of co-products. Second, a new dependent variable appears in the company's maximization problem, which is the level of sale of co-products. Usually companies produce as much as they plan to sell. In the case of joint production, companies lose the ability to determine the optimal level of production of each product. Companies need to choose the level of sale of co-products and provide the possibility of achieving this level.

In summary:

- objective function of a company — aggregate profit;
- maximizing variables — output level of a major product and the levels of sale of co-products;
- constraints — output function according to fixed proportion and conditions that provide the possibility of achieving the optimal level of sale of co-products.

The demarcation between a major product and co-products is based on the profitability of their sale. To compare profitability we need to represent the profitability of each product in one dimension. In the model offered the dimension chosen is the output of one of the products. This means that the demand for all products and corresponding marginal revenue are represented through output of this product. *A major product is a product with the highest marginal revenue at the optimal point.*[3] The output level of such a product then coincides with its sale level. For simplicity take output level of a major product as a dimension.

Consider the maximization problem of a company manufacturing two joint products. One of these products is a major product, and its output is equal to its level of sale. The second product is a co-product, and in particular circumstances its level of sale can differ from its output level.

Assume the following reverse demand functions:

$$P_1 = a_1 - b_1 Q_1 \tag{1}$$

$$P_2 = a_2 - b_2 \rho Q_1, \tag{2}$$

where Q_1 — output of the first product, $\rho \geq 1$ — fixed proportion of production ($Q_2 = \rho Q_1$), P_1, P_2 — prices for the first and the second products, respectively, $a_1, a_2 \geq 0$ — reserve prices for the first and the second products, respectively, $b_1, b_2 \geq 0$ — sensitivities of demand for the first and the second products, respectively.

Then, the marginal revenues from sale of each product have the following form:

$$MR_1(Q_1) = a_1 - 2b_1 Q_1 \tag{3}$$

$$MR_2(Q_1) = \rho a_2 - 2b_2 \rho^2 Q_1 \tag{4}$$

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[3] In this case it is assumed that the relation between marginal revenues corresponds to the relation between gross revenues. There could be the case when they do not correspond to each other due to (a) the correlation of reserve prices, (b) the correlation of demand price elasticities, or (c) the marginal cost.

The first product is a major product if the condition $MR_1(Q_1^*) > MR_2(Q_1^*)$ is satisfied. In such a way, we obtain the first maximization variable—output of a major product ($Q_1 \geq 0$).

The aggregate profit of a company manufacturing joint products consists of revenue from sale of each product minus the cost of production. The latter in the case of joint production can be represented as a function of output of one of products. Representing joint costs as a function of a major product:

$$TC = TC(Q_1) \tag{5}$$

$$MC = c \tag{6}$$

$$\pi = TR_1(Q_1) + TR_2(Q_1) - TC(Q_1) = (a_1 - b_1 Q_1)Q_1 + + (a_2 - b_2 \rho Q_1)\rho Q_1 - TC(Q_1) \tag{7}$$

Maximization of profit with respect to the output of a major product gives the optimal output of a major product and corresponding output of a co-product (see Appendix A).

$$Q_1^* = \frac{a_1 + \rho a_2 - c}{2(b_1 + \rho^2 b_2)} \tag{8}$$

$$Q_2^* = \rho Q_1^* = \rho \frac{a_1 + \rho a_2 - c}{2(b_1 + \rho^2 b_2)} \tag{9}$$

Hence, optimal outputs of products depend on cost of production and parameters of demand functions of both goods and a value of fixed proportion.

The optimal level of sale of a major product always coincides with its output:

$$Q_1'^* = Q_1^* \tag{10}$$

As for a co-product, there could be three cases. In the first case the optimal level of output of the first product is below the level where corresponding marginal revenue from the sale of the second product is equal to zero $\rho Q_1^* < \rho Q_1(MR_2(Q_1) = 0)$ or $Q_1^* < Q_1(MR_2(Q_1) = 0)$. Maximum revenue from sale of the second product, when marginal revenue is strictly greater than zero, is achieved at the lowest marginal revenue. As far as marginal revenue is concerned a decreasing function of the level of sales $(\partial MR/\partial Q_1) < 0$, the optimal level of sale is equal to the maximum possible level that is the whole output. This means that the whole output can be sold with maximum profit:

$$Q_2' = \rho Q_1^* \tag{11}$$

where Q_2'—the volume of second product sales.

The second situation takes place when the optimal output of a major product gives such output of a co-product that marginal revenue of its sale is equal to zero $\rho Q_1^* = \rho Q_1(MR_2(Q_1) = 0)$ or $Q_1^* = Q_1(MR_2(Q_1) = 0)$. In this case the whole output can be sold with maximum profit, which corresponds to (11).

In the third situation the optimal level of output of the first product is above the level where corresponding marginal revenue from the sale of the second product is equal to zero $\rho Q_1^* > \rho Q_1(MR_2(Q_1) = 0)$ or $Q_1^* > Q_1(MR_2(Q_1) = 0)$. It is unprofitable to sell the whole output when marginal revenue is below zero. This implies the appearance of a *glut* (X)—the difference between the level of output and the level sale.

$$X = \rho Q_1^* - Q_2' \tag{12}$$

The level of sale of a co-product that can either coincide or not coincide with its level of output is the second maximization variable. The corresponding constraint is then the following:

$$Q_2' \leq \rho Q_1^* \tag{13}$$

where Q_2'—level of sale of a co-product.

Which of the underlined cases takes place depends on demand parameters (a_1, a_2, b_1, b_2), a value of fixed proportion (ρ) and marginal cost (c) (see Appendix B). The lower the marginal cost, the higher the probability of a glut and the lager is its size. Constant returns to scale (perfectly elastic inverse function of marginal revenue) give an unambiguous effect of demand parameters (a_1, a_2). High demand for a major product (a_1) leads to high probability of a glut; high demand for a co-product (a_2) has the inverse effect. The effect of slopes of demand (b_1, b_2) is uncertain and depends on the relationship between parameters. The same can be said about a value of fixed proportion (ρ).

4. Possible reasons for a glut

The main characteristic of markets of joint products in fixed proportion is a glut, which is the difference between output and sold volume of a product. Output of a major product by definition coincides with the volume of its sales. A part of a co-product called a glut under some conditions is wasted. Assume that wastage of a glut involves no expense and consider possible reasons for its appearance.

As mentioned above, the probability of appearance of a glut and its size depend on demand parameters and marginal cost. Assume that slopes of reverse demand functions are equal ($b_1 = b_2$) and fixed proportion is 1:1.

The first reason for a glut is the market power of a company on both markets. The condition for a glut in this case is the low marginal cost and significant difference in demand for a major product and a co-product ($a_1 > a_2$) and hence in their corresponding marginal revenues.

This situation is represented on the graph below (Fig. 1), where MC—function of marginal cost; MR_1, MR_2—functions of marginal revenue of a major product and a co-product, respectively; MR_{total}—the sum of marginal revenues; D_1, D_2—demand functions. The point of intersection of the marginal cost curve and the total marginal revenue curve takes place on the part of the latter that coincides with the marginal revenue of a major product. The optimal level of output of a major product is equal to Q_1^*. Fixed proportion 1:1 implies that

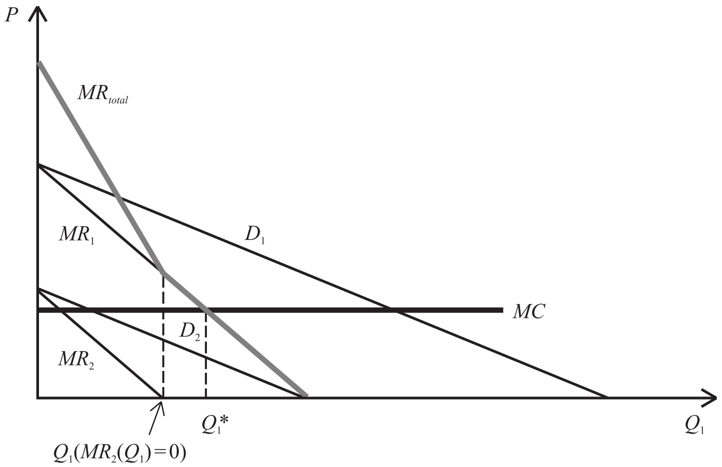

Fig. 1. A glut when there is market power in both markets.

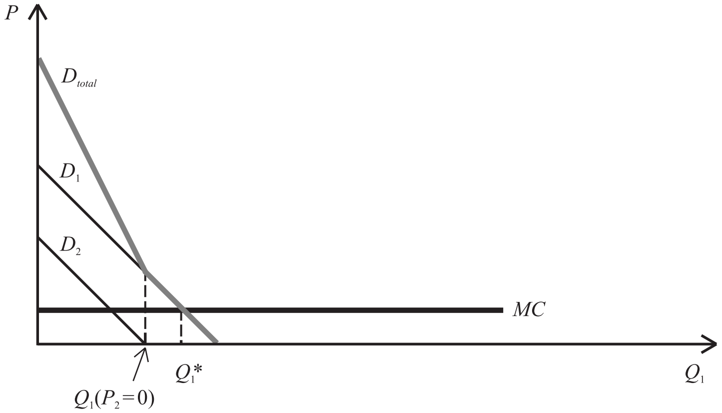

Fig. 2. A glut when both markets are competitive.

output of a co-product is equal to the same amount. The optimal level of a co-product sales is lower than its output and corresponds to the point where its marginal revenue is equal to zero, which is the breakpoint of the total marginal revenue curve $Q_1(MR_2(Q_1) = 0)$. This leads to a glut of a co-product equal to $Q_1^* - Q_1(MR_2(Q_1) = 0)$.

A glut can also appear when both markets are competitive but when demand for a major product significantly exceeds demand for a co-product ($a_1 > a_2$) and marginal cost is low. This case is represented on the Fig. below (Fig. 2), where MC — function of marginal curve; D_1, D_2 — demand functions; D_{total} — the sum of demands. Limited demand for a co-product leads to the situation when sale of this product on the market at a positive price is impossible.

The optimal output of a major product and the corresponding output of a co-product under the fixed proportion 1:1 are equal to Q_1^* that exceeds the amount, which can be sold under nonnegative price $Q_1(P_2 = 0)$. This implies the appearance of the glut equal to $Q_1^* - Q_1(P_2 = 0)$.

Hence, it is necessary to differentiate a glut that is a result of profit maximization when a company possesses market power and a glut that is a result of limited demand when markets are competitive. In the first case, the level of product sales can influence price, and hence under some conditions a company can find it optimal to waste a part of a product. In the second case, a company in principle cannot sell part of a product on the market.

If the assumption that the sensitivities of demand in both markets are equal ($b_1 \neq b_2$) is relaxed, there could be the situation where a product with a higher reserve price has a glut. This situation occurs if the sensitivity of its demand is significantly lower than the sensitivity of demand of the second product ($a_1 \leq a_2$, $b_1 > b_2$). Then, the marginal revenue of such a product under high outputs is equal to zero, while the marginal revenue from sale of the second product is positive. This implies that a product with a lower reserve price and higher sensitivity of demand is a major product, while a product with higher reserve price and lower sensitivity of demand is a co-product.

5. Wastage of a glut

When the wastage of a glut does not carry a cost, its appearance does not influence equilibrium. In this part consider the problem of the wastage of a glut. There could be three situations, i.e., wastage involves no cost (insignificant cost), wastage involves cost (moderate cost), wastage is impossible (prohibitively high cost).

Two cases when a glut can appear were revealed above: (1) it is profitable for a company possessing market power to waste a part of a co-product, (2) it is impossible to sell a part of a co-product due to limited demand for this product on the market.

First, consider the problem of wastage when a glut appears due to the *first reason*. In this case a company has market power and maximizes profit with respect to two variables: output of a major product (Q_1) and the level of sale of a co-product (Q_2'). In addition, the level of sale of a co-product cannot exceed its output level ($Q_2' \leq \rho Q_1$). The demand function of a major product then corresponds to (1), while the demand function of a co-product differs from the demand function (2) and has the following form:

$$P_2 = a_2 - b_2 Q_2' \tag{14}$$

where $a_1, a_2, b_1, b_2, Q_1, Q_2', P_1, P_2 \geq 0$.

Functions of total and marginal costs correspond to equations (5) and (6).

A glut is the difference between the output level of a co-product and its level of sale (12).

Assume that fixed cost of wastage is equal to zero, marginal cost of wastage is constant and equal to d. The profit maximization problem is then the following:

$$\max_{Q_1, Q_2'} \pi = \max_{Q_1, Q_2'} (a_2 - b_2 Q_2') Q_2' + (a_1 - b_1 Q_1) Q_1 - TC(Q_1) - d(\rho Q_1 - Q_2')$$
$$s.t.\ Q_2' \leq \rho Q_1 \tag{15}$$

A Lagrange function and corresponding Kuhn-Tucker conditions let us analyze the outcomes of the optimization problem (see Appendix C) and express a glut through the given parameters (see Appendix D). The result shows that the equation size of a glut inversely depends on marginal cost. The effect of wastage cost is contingent on a fixed proportion and sensitivities of demand.

Consider three situations underlined above: (1) wastage involves no cost ($d = 0$); (2) wastage is impossible (there should be no glut); (3) wastage involves cost ($d > 0$).

The first case occurs when $\lambda = 0$, then $Q_2'^* \leq \rho Q_1^*$. This means that under the particular value of fixed proportion (ρ) output of a co-product and its optimal level of sale coincide; otherwise, the optimal level of sale of a co-product is less than its output, with a glut equal to $(\rho Q_1^* - Q_2'^*) > 0$. In this case, output of a major product corresponds to the point where its marginal revenue is equal to marginal cost and sale of a co-product corresponds to the point where its marginal revenue is equal to zero.

The second case occurs when $\lambda > 0$. A glut is then impossible because $Q_2'^* = \rho Q_1^*$. In this case the optimal output of a major product is less than output that corresponds to the point where its marginal revenue is equal to marginal cost. The optimal level of sale of a co-product is greater than the level that corresponds to the point where marginal revenue of a co-product is equal to zero.

The third situation occurs when wastage is costly, meaning that $d > 0$. In comparison to the first situation, when wastage does not involve cost, the optimal output of a major product is no longer equal to the amount at which marginal revenue is equal to marginal cost, and the optimal level of sale of a co-product does not correspond to the level at which marginal revenue is equal to zero. The optimal output of a major product falls, and hence its optimal price increases. The optimal level of sale of a co-product, in turn, rises; hence, its equilibrium price decreases.

Consider the situation when a glut is impossible in greater detail. The distinctive feature of this situation is that a company moves its focus from a major product to a co-product, wastage of which is impossible. A company produces the amount of a co-product that it can sell on a market with maximum profit, taking into account the profit that it can receive from selling the corresponding output of a major product. This situation is presented above under $\lambda > 0$, when $\rho Q_1^* = Q_2'^*$. For simplicity assume that the fixed proportion is equal to 1:1. If wastage is impossible a company should produce as much as it can sell on the market with maximum profit. This implies that the level of sale of a co-product is no longer a maximization variable in the profit function ($Q_2' = \rho Q_1$). An additional constraint is also needed: a nonnegative price of a co-product to preserve the possibility of selling this product on the market.

$$P_2 = a_2 - b_2 Q_2' \geq 0 \tag{16}$$

$$Q_1 \leq {}^{a_2}\!/_{b_2} \tag{17}$$

The profit maximization problem then has the following form:

$$\max_{Q_1} \pi = \max_{Q_1} (a_1 - b_1 Q_1) Q_1 + (a_2 - b_2 Q_1) Q_1 - TC(Q_1)$$

$$s.t.\ Q_1 = {}^{a_2}\!/_{b_2} \tag{18}$$

Using a Lagrange function and corresponding Kuhn-Tacker conditions (see Appendix C) we get:

$$MR_{total}(Q^*) = c - \lambda \tag{19}$$

We are interested only in cases with gluts. If $\rho = 1$, a glut appears when optimal output of a major product is greater than the output at which the corresponding marginal revenue of a co-product is equal to zero ($Q_1^* > Q_1(MR_2 = 0)$). This gives one more constraint of the maximization problem (see Appendix C).

Thus, if $\lambda = 0$, $\left({}^{a_2}/_{b_2}\right) - Q_1^* \geq 0$. This means that equilibrium corresponds to the point where total marginal revenue is equal to marginal cost if the possibility of selling a co-product on this market is reserved.

If $\lambda > 0$, $Q_1^* = {}^{a_2}/_{b_2}$. Then, outputs of both products fall below the level where total marginal revenue is equal to marginal cost.

If the parameters of demand and cost functions lead to such an output of a major product selling with maximum profit that there is a possibility of selling the corresponding output of a co-product on the market, then this output of a major product is optimal. When wastage is impossible, output and volume of sale of a co-product correspond to the level where its marginal revenue is below zero. Joint costs are taken into account when optimal output of a major product is determined, meaning that when wastage is impossible, the main condition for determining the volume of sale of a co-product is to charge for the latter non-negative price.

If output of a major product that gives the highest profit from its sale corresponds to such an output of a co-product that a part of this output cannot be sold on a market, then the optimal output of a major product is equal to the maximum level at which the corresponding output of a co-product can be sold on the market.

Next, consider the situation when a glut appears due to the second reason, not because a company is seeking additional profit but because it is impossible to sell a part of a co-product on the market. This situation takes place when a company makes a decision under condition of competition. Then, P_1, P_2 are prices of the first and the second good, respectively.

To let a company choose the optimal output assume increasing marginal cost ($\partial MR/\partial Q_1) > 0$).

If there is a glut on the competitive market, a company sells a maximum volume of a co-product that is equal to demand for this product at zero prices (Q_2'). Hence, volume of sale of a co-product is not the maximization variable in this case. The profit function has the following form:

$$\pi = P_1 Q_1 + P_2 Q_2' - TC(Q_1) - d(\rho Q_1 - Q_2') \tag{20}$$

When wastage of a glut does not involve additional cost ($d = 0$),

$$\max_{Q_1} \pi \rightarrow P_1 = MC(Q_1^*) \tag{21}$$

When marginal cost of wastage is positive ($d > 0$),

$$\max_{Q_1} \pi \rightarrow P_1 - MC(Q_1^*) - pd = 0 \tag{22}$$

This can be expressed as $P_1 = MC(Q_1^*) + pd$ or $P_1 - pd = MC(Q_1^*)$. This means that under positive cost of wastage the optimal output of a major product and corresponding output of a co-product fall.

If wastage is impossible, the output of a co-product should be equal to the maximum possible level of sale of a co-product ($\rho Q_1^* = Q_2'$). The optimal output of a major product is then proportional to the maximum level of a co-product that can be sold on the market (see Appendix C).

The model constructed gives an opportunity to make the following suppositions about the markets of chlorine and caustic soda in Russia. First, the technological characteristics of chlorine and caustic soda are such that wastage of both products involves additional cost, and in the case of caustic soda, the cost is prohibitively high. According to the model, if there is a glut of caustic soda, then its output level should correspond to its optimal level of sale. This leads to the situation when the real optimal output of chlorine is lower than its optimal output as if peculiarities of chlorine's production and impossibility of caustic's wastage are not taken into account; the real optimal level of sale of caustic is higher than the corresponding optimal level. This rule is valid either if a company has market power or is under competitive conditions.

Second, if there is a glut of chlorine, then a company needs to take into account the cost of wastage of chlorine when it determines its optimal level of output and sale. Then, the real optimal output of caustic soda is lower than the optimal output that does not take into account peculiarities of production and cost of wastage of chlorine. In turn, the real optimal output of chlorine is higher than the corresponding optimal level because an alternative for sale of the latter is wastage that involves additional cost.

Third, an additional observation is the fact that if the market of caustic soda is competitive, then for a company to make a decision to sell this product on the market its price should be higher than the marginal cost because in addition to the latter, the unavoidable cost of wastage of chlorine is taken into account.

6. Separate processing and an open economy

In the analysis above the only way to let output and the level of sale of a co-product to differ is the wastage of a glut. The possibility of separate processing or sale of a product on a distant market (including export) adds a new market to the model. In this analysis a new market is considered as a way to dispose of a glut. An "old" market then is an objective market meaning that a company sells with maximum profit both a major product and a co-product on this market and in the case of a glut may separately process and sell it on a different market or transport and sell it on a distinct market. Both ways involves additional costs. In the first case, cost comes from separate processing, and in the second one — from transportation. These costs have a similar effect on market equilibrium and are considered jointly. Assume that the marginal costs of separate processing or transportation are constant and equal to f (fixed cost of separate processing and transportation of a unit of a glut are equal to zero).

The possibility of selling a glut on a new market adds a new maximization variable to the profit maximization problem: a volume of a co-product that is separately processed or transported to a distant market (Q_2''). As far as the possibility of selling a co-product on a new market is considered only as a way to dispose of a glut, the volume of sale of a co-product on a new market cannot exceed a glut.

$$\rho Q_1 - Q_2' \geq Q_2'' \tag{23}$$

This condition is stricter than $Q_2' \leq \rho Q_1$, and hence if it holds, the later holds automatically.

Consider different market structures on new and "old" markets. To keep an "old" market objective a company should possess no less market power on it in comparison to a new market. Thus assume that a company has market power on an "old" market and consider two cases.

1. *A company has market power both on an "old" market and on a new market.* The demand functions on an "old" market then correspond to equations (15), (1) and the demand function for a co-product on a new market has the following form:

$$P_2'' = a_2' - b_2' Q_2'' \tag{24}$$

where $a_2', b_2', Q_2'', P_2'' \geq 0$.

A glut is equal to the difference between output and the level of sales of a co-product (see 12).

For simplicity assume that a fixed proportion is equal to 1:1 ($\rho = 1$).

$$\begin{aligned} \max_{Q_1, Q_2', Q_2''} \pi = \max_{Q_1, Q_2', Q_2''} & (a_2 - b_2 Q_2')Q_2' + (a_1 - b_1 Q_1)Q_1 - TC(Q_1) - \\ & - d(Q_1 - Q_2' - Q_2'') + (a_2' - b_2' Q_2'')Q_2'' - fQ_2'' \\ s.t.\ & Q_1 - Q_2' \geq Q_2'' \end{aligned} \tag{25}$$

Using a Lagrange function and corresponding Kuhn-Tucker conditions consider three cases: (1) wastage involves no cost ($d = 0$), (2) wastage involves cost ($d > 0$), (3) wastage is impossible (there should be no glut) (see Appendix C).

If $\lambda = 0$, then $Q_1^* - Q_2'^* \geq Q_2''^*$. In the first case $d = 0$. Then, a company separately processes or transports such amount of a co-product that marginal revenue from its sale on a new market is equal to the cost of separate processing or transportation.

In the second case, $d > 0$. This implies that a company is ready to sell a glut on a new market even if marginal revenue is less than the marginal cost of separate processing or transportation.

If $\lambda > 0$, then $Q_1^* - Q_2'^* = Q_2''^*$. This corresponds to the third case when wastage is impossible. The optimal volume of sale of a glut is then higher than the volume that corresponds to equality of marginal revenue and marginal cost of separate processing or transportation. In this case the optimal output of a major product falls and the optimal level of sale of a co-product rises.

2. *A new market is competitive, and on an "old" market a company has market power.* Then, P_2' is the price of a co-product on a new market.

$$\begin{aligned} \max_{Q_1, Q_2', Q_2''} \pi = \max_{Q_1, Q_2', Q_2''} & (a_2 - b_2 Q_2')Q_2' + (a_1 - b_1 Q_1)Q_1 - TC(Q_1) - \\ & - d(Q_1 - Q_2' - Q_2'') + P_2' Q_2'' - fQ_2'' \\ s.t.\ & Q_1 - Q_2' \geq Q_2'' \end{aligned} \tag{26}$$

Again, consider three cases using a Lagrange function and Kuhn-Tucker conditions (see Appendix C).

If $\lambda = 0$, then $Q_1^* - Q_2'^* \geq Q_2''^*$. In the first case $d = 0$. The company does not then separately process or transport a glut if price on a new market is lower than the marginal cost of separate processing or transportation.

In the second case, $d > 0$. The acceptable price for a company to sell a part of a glut or the whole glut on a new market then positively relates to the cost of separate processing or transportation and negatively relates to the cost of wastage.

If $\lambda > 0$, then $Q_1^* - Q_2'^* = Q_2''^*$. This corresponds to the third case when wastage is impossible. The acceptable price for a company to sell the whole glut then falls under the marginal cost of separate processing or transportation. This in turn explains the phenomenon when a company exports a product bearing losses. If it is impossible to sell a glut on a new market at a positive price ($Q_2'' = 0$), output of a major product should be equal to the volume where corresponding output of a co-product can be sold on an "old" market ($Q_1^* = Q_2'^*$).

If a new market is competitive, a company has a choice in the first two cases, when wastage is possible: to sell a glut or not. When the wastage of a glut is impossible, a company has to sell it at any price. As far as negative price is not taken into account and means that a company cannot sell a glut on this market, a company should sell the whole output on an "old" market and $Q_1^* = Q_2'^*$, hence $Q_2''^* = 0$.

3. There also could be the case when *both markets are competitive.* The volume of sale of a co-product on an "old" market then is not a maximizing variable and profit-maximizing function has the following form:

$$\max_{Q_1, Q_2''} \pi = \max_{Q_1, Q_2''} (P_1 Q_1 + P_2' Q_2' - TC(Q_1) - d(Q_1 - Q_2' - Q_2'') + P_2' Q_2'' - f Q_2''$$

$$s.t.\ Q_1 - Q_2' \geq Q_2'' \tag{27}$$

On the basis of a corresponding Lagrange function and Kuhn-Tucker conditions it is possible to make the following conclusions according to the three cases underlined above (see Appendix C).

If $\lambda = 0$, $Q_1 - Q_2' \geq Q_2''$. The price of a major product should cover cost of production if $d = 0$, and in addition the cost of wastage if $d > 0$. A glut is then sold on a new market if its price is equal to the cost of separate processing or transportation if $d = 0$ and can be even lower if $d > 0$.

If $\lambda > 0$, wastage is impossible and $Q_1^* - Q_2' - Q_2''^* = 0$. This implies that a glut ($Q_2''^* = Q_1^* - Q_2'$) is sold at any positive price even below the cost of separate processing or transportation. If a co-product cannot be sold on a new market ($Q_2'' = 0$) at a positive price, this turns us back to the situation when output of a major product is determined according to the possibility of selling a co-product on the market ($Q_1'' = Q_2'$). Output of a major product then falls and, consequently, its price is above the marginal cost of production.

The model's extension by adding a new market lets us analyze the relationship of alternative strategies of a company with respect to a glut, in particular: sale of higher volumes of a co-product on an "old" market, wastage of a glut and sale of a glut on a new market.

Return to the example with chlorine and caustic soda. Technological characteristics of chlorine imply that it should be sold within three days after production. This means that transportation of chlorine to distant markets is impossible. However, it can be separately processed into chlorinated derivatives (PVC), which offers an opportunity to sell a glut of chlorine on a new market (downstream market) of PVC. Transportation of caustic soda is possible, but it involves moderate cost that enables sale of a glut of caustic soda on a distinct market. According to the model, if wastage is impossible, as it is in the case of caustic soda, a company agrees to sell a glut even at a price below the cost of transportation both if the company has market power on a new market and under competitive conditions. Thus, *sale of caustic at a price that from first glance is unjustifiable with respect to domestic price is not the consequence of more intensive competition on a foreign market, but is the way of selling a glut.* If sale of caustic on a new market is impossible, the price of chlorine exceeds the marginal costs of production even under competitive conditions on the market of chlorine.

If there is a glut of chlorine, then a company's decision about its separate processing depends on the market structure of chlorinated derivatives and hence on the price at which a company can sell its product on this market because there is an alternative in the form of wastage in the case of chlorine. If the market of chlorinated derivatives is highly competitive, then the Russian company cannot influence the price on this market. However, a company decides to separately process chlorine even if the price on the market of chlorinated derivatives is below the marginal cost of separate processing because cost of wastage that a company incurs otherwise is taken into account. However, if sale of chlorine on a new market at a positive price is impossible, the price of caustic on an "old" market increases because its output falls. If the Russian company has market power on the market for PVC, it separately processes a higher volume of chlorine than the profit maximization conditions of this market demand because the cost of wastage is taken into account.

Thus, in addition to joint costs of production and demand for each product the factor that determines equilibrium is cost of wastage. The higher the cost of wastage, the higher the probability that for a company it is profitable to sell a glut and hence the lower its price on a new market. Other factors are costs of separate processing and transportation. Then, a company needs to choose between separate processing (transportation) and sale at the split-off point.

Hence, optimal outputs, the levels of sale and prices of joint products before and after separate processing or transportation can differ from their optimal values if the markets of each product are considered separately disregarding negative externalities connected with a glut, of which wastage involves costs.

7. Conclusion

Even on the basis of basic microeconomic analysis, disregarding a company's internal organization and economic analysis of contractual relations, it is shown that prices and the levels of sale of joint products are different from these parameters as if there is no joint production, cost of wastage and the possibility of separate processing and transportation.

If the antitrust authority investigates markets of joint products, it should take into account the peculiarities of these markets (especially in the absence of direct evidence). Otherwise, this could result in wrong conclusions about the nature of companies' behavior and hence errors of law enforcement. The engagement of experts of each industry under investigation who can provide comprehensive analysis of their functioning is important in regard to increasing the efficiency of antitrust policy tools (Schmalensee, 2012). This could improve the balance of the type 1 and the type 2 errors in law enforcement practice.

Appendix A

$$\pi = TR_1(Q_1) + TR_2(Q_1) - TC(Q_1) = (a_1 - b_1Q_1)Q_1 + \\ + (a_2 - b_2\rho Q_1)\rho Q_1 - TC(Q_1) \tag{A.1}$$

$$\max_{Q_1} \pi \rightarrow a_1 - 2b_1Q_1^* + \rho a_2 - 2b_2\rho^2Q_1^* - c = 0 \tag{A.2}$$

$$MR_1(Q_1^*) + MR_2(Q_1^*) - c = 0 \tag{A.3}$$

$$MR_{total}(Q_1^*) = c \tag{A.4}$$

Optimal output of the first (major) product:

$$Q_1^* = \frac{a_1 + \rho a_2 - c}{2(b_1 + \rho^2 b_2)} \tag{A.5}$$

Corresponding output of the second product (co-product):

$$Q_2^* = \rho Q_1^* = \rho\frac{a_1 + \rho a_2 - c}{2(b_1 + \rho^2 b_2)} \tag{A.6}$$

Appendix B

$$Y = Q_1^* - Q_1(MR_2(Q_1) = 0 \tag{B.1}$$

The higher Y is, the higher the probability of a glut and its size is.

$$MR_2(Q_1) = 0 \tag{B.2}$$

$$\rho a_2 - 2b_2\rho^2Q_1 = 0 \tag{B.3}$$

$$Q_1 = \frac{a_2}{2b_2\rho} \tag{B.4}$$

According to equation (C.1):

$$Q_1^* = \frac{a_1 + \rho a_2 - c}{2(b_1 + \rho^2 b_2)} \tag{B.5}$$

Then,

$$Y = \frac{a_1 + \rho a_2 - c}{2(b_1 + \rho^2 b_2)} - \frac{a_2}{2b_2\rho} = \frac{a_1b_2\rho - b_1a_2 - b_2c\rho}{2b_1b_2\rho + 2\rho^2 b_2} \tag{B.6}$$

$$\frac{\partial Y}{\partial c} \leq 0, \quad \frac{\partial Y}{\partial a_1} \geq 0, \quad \frac{\partial Y}{\partial a_2} \leq 0 \tag{B.7}$$

Appendix C

Kuhn-Tucker conditions	Company has market power		Market is competitive		There is a possibility of separate processing and economy is open		
	Wastage of a glut	Wastage of a glut is impossible	Wastage of a glut	Wastage of a glut is impossible	Company has market power both on new and "old" markets	A new market is competitive, on an "old" market a company has market power	Both markets are competitive
Lagrange function	$L = \pi + \lambda(Q_2' - \rho Q_1)$ where $\lambda \geq 0$	$L = \pi + \lambda(Q_1 - a_2/b_2)$ where $\lambda \geq 0$			$L = \pi + \lambda(Q_2' - \rho Q_1)$ where $\lambda \geq 0$	$L = \pi + \lambda(Q_2' - \rho Q_1)$ where $\lambda \geq 0$	$L = \pi + \lambda(Q_2' - \rho Q_1)$ where $\lambda \geq 0$
$\frac{\partial L}{\partial Q_1}$	$a_1 - 2b_1 Q_1^* - c - - d\rho - \rho\lambda = 0$	$a_1 + a_2 - (2b_1 + b_2)Q_1^* - c + + \lambda = 0$			$a_1 - 2b_1 Q_1^* - c - - d - \lambda = 0$	$a_1 - 2b_1 Q_1^* - c - - d - \lambda = 0$	$P_1 - MC(Q_1^*) - - d - \lambda = 0$
$MR_1(Q_1^*)$ or P_1	$c + d\rho + \rho\lambda$	$c - \lambda$	$P_1 = MC(Q_1^*) + \rho d$		$c + d + \lambda$	$c + d + \lambda$	$MC(Q_1^*) + d + \lambda$
Q_1^*	$\frac{a_1 - c - d\rho - \rho\lambda}{2b_1}$	$\frac{a_1 - c - \lambda}{2b_1}$		$Q_1^* = \frac{Q_1'}{\rho}$	$\frac{a_1 - c - d - \lambda}{2b_1}$	$\frac{a_1 - c - d - \lambda}{2b_1}$	
$\frac{\partial L}{\partial Q_2'}$	$a_2 - 2b_2 Q_2'^* - - d + \lambda = 0$				$a_2 - 2b_2 Q_2'^* + + d + \lambda = 0$	$a_2 - 2b_2 Q_2'^* + + d + \lambda = 0$	
$MR_2(Q_2'^*)$	$d - \lambda$				$d - \lambda$	$d - \lambda$	
$Q_2'^*$	$\frac{a_2 + d + \lambda}{2b_2}$				$\frac{a_2 + d + \lambda}{2b_2}$	$\frac{a_2 + d + \lambda}{2b_2}$	
$\frac{\partial L}{\partial Q_2''}$					$d + a_2' - 2b_2 Q_2''^* - - f + \lambda = 0$	$d + P_2' - f + \lambda = 0$	$-f + P_2' + d + \lambda = 0$
$MR_2'(Q_2''^*)$ or P_2'					$f - d + \lambda$	$f - d - \lambda$	$f - d - \lambda$
$Q_2''^*$					$\frac{d + a_2' - f + \lambda}{2b_2'}$		
$\frac{\partial L}{\partial \lambda}$	$\rho Q_1^* - Q_2'^* \geq 0$ $\lambda(\rho Q_1^* - Q_2'^*) = 0$ $\lambda \geq 0$	$a_2/b_2 - Q_1^* \geq 0$ $\lambda(a_2/b_2 - Q_1^*) = 0$ $\lambda \geq 0$			$Q_1^* - Q_2'^* - Q_2''^* \geq 0$ $\lambda(\rho Q_1^* - Q_2'^* - Q_2''^*) = 0$ $\lambda \geq 0$	$Q_1^* - Q_2'^* - Q_2''^* \geq 0$ $\lambda(\rho Q_1^* - Q_2'^* - Q_2''^*) = 0$ $\lambda \geq 0$	$Q_1^* - Q_2'^* - Q_2''^* \geq 0$ $\lambda(\rho Q_1^* - Q_2'^* - Q_2''^*) = 0$ $\lambda \geq 0$

Appendix D

$$X = \rho \frac{a_1 - c - d\rho - \rho\lambda}{2b_1} - \frac{a_2 + d + \lambda}{2b_2} \tag{D.1}$$

$$\frac{\partial X}{\partial d} = -\frac{\rho}{2b_1} - \frac{1}{2b_2} \tag{D.2}$$

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