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Empfohlene Zitierung / Suggested Citation:
Ertl, M., & Rabitsch, K. (2024). *R* and Convergence.* (IHS Working Paper, 55). Wien: Institut für Höhere Studien (IHS), Wien. https://nbn-resolving.org/urn:nbn:de:0168-ssoar-94362-2

IHS Working Paper 55
May 2024

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Title

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R* and Convergence

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May 12, 2024

Abstract

We explore the natural rate of interest, shortly r^*, in emerging economies. If economic growth originates from convergence, then growth, say, from technological progress will be lower than we find in the data and, hence, r^* will be lower. Ignoring convergence upwardly biases our estimates of r^*. We extend the New Keynesian small open economy model to take account of convergence. The model is estimated with Bayesian techniques for four emerging economies in Central and Eastern Europe: Poland, Czech Republic, Hungary and Romania. The estimation process is informed by empirical evidence about a rapid catch-up of our example economies during the period from 2003 to 2019. We confirm the decline in r^* over the last decades. When we account for capital deepening, we find meaningful differences with non-negligible implications for monetary policy.

Keywords: natural rate of interest; convergence; New Keynesian DSGE model; Central and Eastern Europe
JEL classification: E3, E4, E5

Declarations:

All the authors have accepted responsibility for the entire content of this submitted manuscript and approved submission. The authors declare no conflicts of interest regarding this article.

Acknowledgements:

The authors would like to thank Kathrin Gruber, Helmut Hofer, Robert M. Kunst, Michael Reiter, Adrian Wende and the participants of 2023 Meeting of the Austrian Economic Association (NOeG) for helpful comments that have led to improvement of the paper. The authors gratefully acknowledge help from Franz X. Zobl for the empirical evidence section.

1 Introduction

Neoclassical growth theory predicts that economic convergence across countries arises from capital deepening. Economies that have less capital per worker (relative to their long-run capital per worker) tend to have higher growth rates and higher rates of return; a property that derives from the assumption of diminishing marginal returns to capital. Capital accumulation drives down the rate of return and, hence, the real rate of interest. As an economy develops, there is a tendency for the real rate of interest to fall over time. In a business cycle framework such as the New Keynesian model, the real interest rate of the counterfactual economy without shocks and frictions is often referred to as the natural rate of interest, or Wicksellian rate or just r^*. This paper investigates the relationship between economic convergence and the natural rate of interest.

The accession to the European Union (EU) during the 2000s of emerging economies in Central and Eastern Europe (CEE) and the subsequent decade is a historical episode of rapid convergence that allows us to study the interaction between economic catch-up and the natural rate of interest. For this purpose, we present empirical evidence of beta convergence in a panel of European countries. Robust estimates of beta, in line with the iron law of convergence (Barro, 2012), inform our approximate level of the capital stocks of EU accession countries in the years after the end of the iron curtain.

We estimate the natural rate of interest in a New Keynesian small open economy model with capital accumulation. The Bayesian model estimation takes into account the transitional dynamics for the period between the years 2003 and 2019 for four emerging economies in CEE: Poland, Czech Republic, Hungary and Romania. Recently, it has been found that the open economy framework provides a better fit to the data than the closed economy counterpart (Zhang et al., 2021). Our four example economies are significantly open to international trade, foremost with the Euro Area, but small enough that domestic shocks or policies would not influence the world economy. While r^* has frequently been considered in an open economy set-up (Mesonnier and Renne, 2007; Berger and Kempa, 2014; Pedersen, 2015; Wynne and Zhang, 2018), so far emerging economies in CEE are underrepresented (Grafe et al., 2018; Hledik and Vlček, 2018; Arena et al., 2020; Kupkovič et al., 2020).

Our exploration of economic convergence suggests a new perspective towards the secular drivers of r^*. While the literature about the natural rate of interest has grown rapidly during the last decade, the research focus has mostly been on the secular decline of r^* in advanced economies and its drivers.[1] The long-term decline of global real interest rates coincides with a fall in trend growth (Laubach and Williams, 2003; Del Negro et al., 2017, 2019; Gagnon et al., 2021; Holston et al., 2021). Demographic transition is a main determinant of the long-term decline in r^* (Kara and von Thadden, 2014; Carvalho et al., 2016; Lisack et al., 2017; Bielecki et al., 2020; Gagnon et al., 2021). Rachel and Smith (2017) take into account several drivers of the decline, including demographic changes, rising inequality, increased saving in emerging markets, the fall in the relative price of capital goods and lower demand for public investment. Summers (2014) notion of a new area of secular stagnation invoked research on the decline in r^* (Eggertsson et al., 2019; Rachel and Summers, 2019). Secular stagnation, coined originally by Hansen (1939), is associated with a reduction of investment demand, a declining rate of population growth, slowing technological progress, changes in the income distribution and a substantial decline or even permanently negative equilibrium rate of interest. Other branches in the literature emphasize a shortage in safe assets (Kirshnamurthy and Vissing-Jorgensen, 2012; Del Negro et al., 2017; Neri and Gerali, 2019; Ferreira and Shousha, 2023) or the 'global savings glut' resulting from an increase in desired savings and demand for safe assets in emerging economies (Bernanke, 2005; Bernanke et al., 2011). Taking stock of the prolonged periods of expansionary monetary policies in many economies, researchers investigate how (in contrast to the standard New Keynesian model) monetary policy can influence productivity growth and, hence, the natural rate of interest (Aghion et al., 2019; Bergeaud et al., 2019). Other, newer strands in the literature investigate the interaction of r^* and market concentration (Liu et al., 2022), wealth inequality (Fernández-Villaverde et al., 2021) or capital misallocation (Monacelli et al., 2018; Acharya et al., 2021; Asriyan et al., 2021; González et al., 2021).

The rest of the paper is organized as follows: Section 2 provides introductory empirical evidence of economic convergence. In section 3 we derive the New Keynesian small open

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[1] A comprehensive overview about estimates of r^* and its drivers can be found, for example, in Brand et al. (2018).

economy framework. Section 4 describes the empirical estimation of the model. and section 5 present the results and discussion. The last section concludes.

2 Evidence of convergence

In this chapter, we provide evidence of economic convergence in Europe since the late 1990s. In the neoclassical growth model (Solow, 1956) convergence results from capital deepening. Capital accumulation leads to higher capital per worker and to a decrease of the marginal product of capital. The marginal product of capital equals the real interest rate, which equals r^* in the frictionless growth model. Hence, the convergence process coincides with a decline in r^*. Figure 1 shows facts about convergence in the four emerging economies in CEE, which we investigate: Poland, Czech Republic, Hungary and Romania. Since the turn of the millenium, the four countries show a substantial catch-up in real per capita gross domestic product (GDP). Between 1995 and 2019, real per capita GDP converged to Germany's GDP per capita by 30 % in Poland, 19 % in the Czech Republic, 19 % in Hungary and 24 % in Romania.

Next, we estimate growth regressions for a panel of European countries. Recently, Kremer et al. (2022) revisited the empirical tests of convergence in a global sample and cast doubt on the disillusion about convergence, which had spread during the 1990s. At this time, the data rather spoke for divergence in incomes across countries and time (see, for example Barro, 1991). Now that there are additional 25 years of economic development, Kremer et al. (2022) show evidence in favor of convergence, particularly after the turn of the millenium.

In table 1, we add evidence for a panel of 42 European countries and four periods, each of which spans five years starting in 1997 and ending in 2017.[2] We find that unconditional (absolute) convergence is broadly consistent with the "iron law" of convergence of 2 % per year (Barro, 2012). The unconditional convergence coefficient in specification (1) is 1.5 %

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[2] We follow the most standard approach, testing for beta convergence by estimating OLS-regressions with fixed effects for periods (5-year), clustered at the country level, see Durlauf et al. (2005). The dependent variable is the average annual GDP per capita growth rate of the subsequent five-year period of the log GDP per capita variable. A detailed description of the sample and the definition and the sources of the variables is included in Appendix A.1.

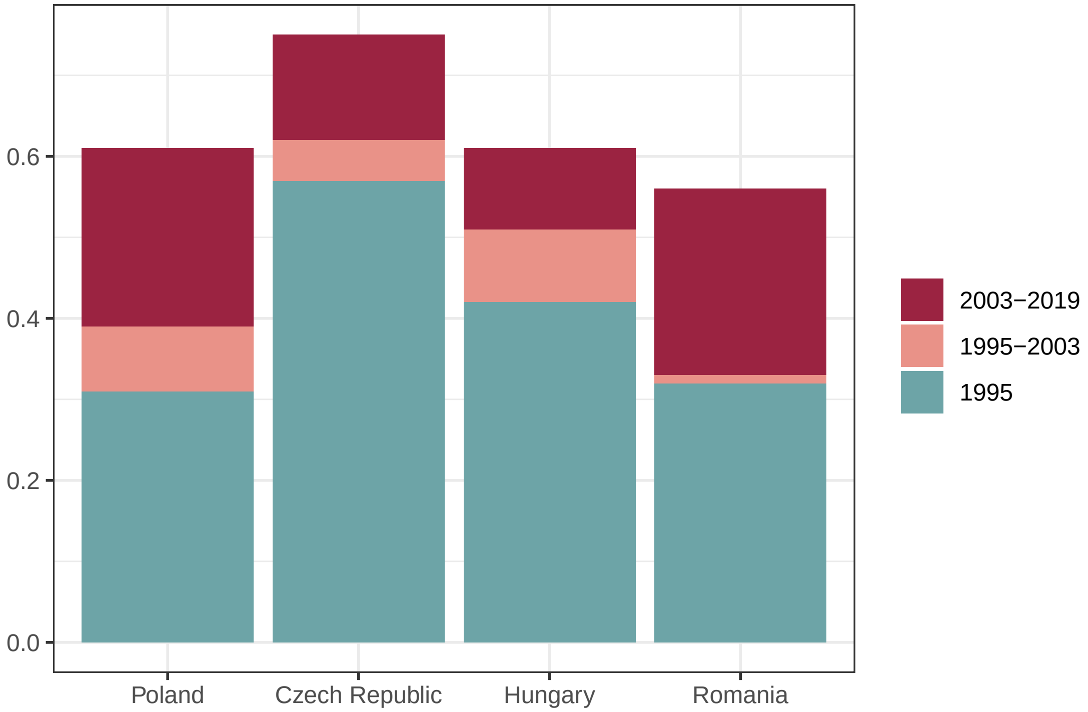

Figure 1: Convergence. The graph depicts real GDP per capita at purchasing power parities relative to Germany' GDP p. c.. Source: IMF World Economic Outlook (2021).

and statistically significant. Specification (2) shows that convergence was fastest during the Eastern EU-enlargement between the years 2002 and 2007 (Poland, Czech Republic, and Hungary joined the EU in 2004 and Romania in 2007), when the convergence coefficient increases to 2.2 % (adding up 0.9 % and 1.3 %). It remains statistically significant and sizeable at 1.8 % (adding up 0.9 % and 0.9 %) during the subsequent five years (2008-2012). Specification (3) reveals that annual absolute convergence was of the magnitude of 2.3 % for EU countries (summing up 1.2 % and 1.1 %). Compared to Kremer et al. (2022), who find a coefficient of 0.7 % between 2005 and 2015 in a global sample, unconditional convergence in Europe appears higher for that time.

The specifications (4) to (6) in table 1 show evidence of conditional convergence. In specification (4), the Solow fundamentals, i. e. the average annual investment rate, the population growth rate and the growth rate of the labor force, are added to the regression. Higher investment and more rapid growth of the labor force are positively (and statistically significantly) associated with GDP per capita growth while population growth shows a

Table 1: Absolute and conditional convergence in Europe between 1997 and 2017

dep. var.: GDP p. c. growth	(1)	(2)	(3)	(4)	(5)	(6)
Log GDP p.c.	-0.015***	-0.009*	-0.012***	-0.017***	-0.021***	-0.020***
	(0.003)	(0.005)	(0.003)	(0.003)	(0.007)	(0.007)
2003-2007	0.012***	0.145***	0.013***	0.012***	0.009**	0.011**
	(0.004)	(0.050)	(0.004)	(0.004)	(0.004)	(0.005)
2008-2012	-0.027***	0.063	-0.025***	-0.028***	-0.032***	-0.026***
	(0.004)	(0.049)	(0.004)	(0.004)	(0.005)	(0.005)
2013-2017	-0.006*	-0.036	-0.005	-0.006	-0.011**	-0.004
	(0.003)	(0.070)	(0.004)	(0.004)	(0.005)	(0.006)
2003-2007 × Log GDP p.c.		-0.013***				
		(0.005)				
2008-2012 × Log GDP p.c.		-0.009*				
		(0.005)				
2013-2017 × Log GDP p.c.		0.003				
		(0.007)				
EU × Log GDP p.c.			-0.011*			
			(0.006)			
EU			0.117*			
			(0.064)			
Investment				0.001**	0.001**	0.001**
				(0.000)	(0.000)	(0.000)
Population				-0.007**	-0.005	-0.005
				(0.003)	(0.003)	(0.004)
Labor Force				0.002*	0.003*	0.004
				(0.001)	(0.001)	(0.002)
Institutions					-0.002	0.004
					(0.004)	(0.005)
Schooling					0.003**	0.002*
					(0.001)	(0.001)
Openness					0.000	-0.000
					(0.000)	(0.000)
Credit						-0.000**
						(0.000)
FDI						0.000***
						(0.000)
Public debt						-0.000*
						(0.000)
Constant	0.176***	0.123**	0.147***	0.182***	0.193***	0.204***
	(0.028)	(0.051)	(0.030)	(0.030)	(0.070)	(0.065)
Observations	166	166	166	152	150	135

Cluster-robust standard errors are reported in parentheses. Time fixed effects. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
Sources: own calculations, data from IMF WEO and IFS, Penn World Table, World Bank, Barro and Lee (2013).

negative correlation. Adding these control variables has a slightly positive effect on the convergence coefficient, increasing to 1.7 % and remaining statistically significant. We add further growth correlates in specifications (5) and (6) including annual averages of a proxy for institutions, years of schooling, trade openness, growth of credit to the private sector, foreign direct investment (FDI) and public debt to GDP. Years of schooling and FDI are positively and stastically significantly associated with p. c. GDP growth. High public debt ratios are associated with lower GDP p. c. growth. Credit to the private sector is negatively correlated with GDP per capita growth, which might be due to the fact that credit to GDP ratios are higher in more advanced countries as well as Southern European countries. The measures of institutional quality and trade openness are not found to be statistically significant. In both specifications, the convergence coefficient is statistically significant and around 2.0 %. Additionally, we find evidence that conditional convergence was fastest during the Eastern enlargement period between 2002 to 2007 (convergence coefficient of 2.9 %) and particularly fast within the EU respectively for EU member countries in CEE (see appendix A.1). Since we have now presented some evidence of economic convergence, we return to the analysis of the natural rate of interest in a model framework and estimation procedure allowing for convergence.

3 The New Keynesian small open economy model

We consider a model of a small open economy with staggered price setting that builds on the framework developed by Galí and Monacelli (2005). In this framework, the size of the home economy is negligible relative to that of the world economy, therefore world aggregates (output, inflation) can be taken as exogenous. We extend the model by capital accumulation that is subject to adjustment costs (see Galí and Monacelli (2016)). A similar framework for studying the natural rate of interest but without capital accumulation is derived in Zhang et al. (2021). We incorporate a preference shock, a markup shock, a terms of trade shock and a non-stationary shock process for technological progress as in Zhang et al. (2021); apart from a standard monetary policy shock. Additionally, our model features an investment shock as in Justiniano et al. (2010), Justiniano et al. (2011)), which has been found recently to play an important role also in emerging economies (Dogan, 2019).

For simplicity, we stick to a version of the small open economy model in which the law of one price holds under producer currency pricing. For a richer small open economy model featuring imperfect pass-through, Caraiani (2013) finds that the estimation results for CEE are generally similar. Furthermore, we assume complete international assets markets, for which McKnight et al. (2020) find favorable evidence among emerging economies in Latin America. The Bayesian estimation is described in the subsequent section 4.

3.1 Domestic households

The small open economy is inhabited by a representative household who solves the lifetime utility maximization problem

$$E_0 \sum_{t=0}^{\infty} \beta^t e^{-g_t} \left[\frac{C_t^{1-\frac{1}{\tau}}}{1-\frac{1}{\tau}} - \frac{N_t^{1+\varphi}}{1+\varphi} \right], \tag{1}$$

where N_t are the hours worked and C_t is the aggregate consumption index,

$$C_t \equiv \left[(1-\gamma)^{\frac{1}{\eta}} \left(C_{H,t}\right)^{\frac{\eta-1}{\eta}} + \gamma \left(C_{F,t}\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{2}$$

and $C_{H,t}$ is a composite of differentiated domestic goods,

$$C_{H,t} \equiv \left(\int_0^1 C_{H,t}\left(i\right)^{\frac{\epsilon_t-1}{\epsilon_t}} di \right)^{\frac{\epsilon_t}{\epsilon_t-1}}, \tag{3}$$

and $C_{F,t}$ an analogous composite of imported goods, where $i \in [0,\, 1]$ denotes the home goods varieties. The parameter τ is the elasticity of intertemporal substitution, $\varphi > 0$ denotes the inverse of the Frisch elasticity of labor supply, $\eta > 0$ is the substitution between home and foreign goods, $0 < \gamma < 1$ is a measure for the degree of openness, and $\epsilon_t > 1$ is the elasticity of substitution between product varieties. The discount factor is denoted by $0 < \beta < 1$.[3] The investment index, I_t, can be defined in analogy to the consumption index

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[3] In order to avoid misunderstanding, the beta referring to convergence of the growth regressions in section 2 is written out, the discount factor is the greek letter, β, and the Beta distribution, appearing later, is written with a capital.

(with identical weights and substitution elasticities). An exogenous preference shock g_t is introduced following a stationary AR(1) process:

$$g_t = \rho_g g_{t-1} + \varepsilon_{g,\,t},\ \varepsilon_{g,\,t} \sim N\left(0,\, \sigma_g\right), \tag{4}$$

where $-1 < \rho_g < 1$ and σ_g measures persistence and volatility. The representative household invests in financial assets and accumulates physical capital, which it rents out to firms. The household's optimization problem is subject to the budget constraint that is

$$P_t C_t + P_t I_t + E_t\left(\Lambda_{t,\,t+1} D_{t+1}\right) \leq W_t N_t + R_t^K K_{t-1} + D_t + T_t + Pr_t. \tag{5}$$

where $P_t \equiv \left[\left(1-\gamma\right)\left(P_{H,\,t}\right)^{1-\eta} + \gamma\left(P_{F,\,t}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}$ is the price level of the domestic bundle of consumption goods, C_t, and $P_{H,\,t}$ and $P_{F,\,t}$ are the sub-indices of the domestically-produced and imported goods. I_t is an equivalent bundle of investment goods, W_t is the nominal wage, and Pr_t denotes the profits accrued from the ownership of the domestic firms. The nominal holdings of Arrow-Debreu securities are denoted by D_t and $\Lambda_{t,\,t+1}$ is the state-contingent price of the security. K_{t-1} denotes capital and R_t^K is the rental rate of capital. T_t is a lump-sum tax raised by the domestic government. The accumulation of capital subject to investment adjustment costs obeys

$$K_t = \left(1-\delta\right) K_{t-1} + I_t \nu_t \left[1 - \frac{\kappa_I}{2}\left(\frac{I_t}{I_{t-1}} - \widetilde{\Gamma}_A\right)^2\right], \tag{6}$$

where $\widetilde{\Gamma}_A = \Gamma_A^{\frac{\tau+\tau\varphi}{1+\tau\varphi}}$, which is the gross steady-state growth rate of productivity. Investment in a final composite good is denoted by I_t, $0 \leq \delta \leq 1$ is the rate of physical depreciation, and $\kappa_I \geq 0$ is a parameter that scales the size of (quadratic) adjustment costs in investment. Additionally, we introduce an investment shock, ν_t, as in Justiniano et al. (2010, 2011). It is a shock to the marginal efficiency of investment and represents an exogenous disturbance to the process by which investment goods are transformed into installed capital or technological factors specific to the production of investment goods. It follows an AR(1) process,

$$\nu_t = \rho_\nu \nu_{t-1} + \varepsilon_{\nu,\,t},\ \varepsilon_{\nu,\,t} \sim N\left(0,\, \sigma_\nu\right), \tag{7}$$

where $-1 < \rho_\nu < 1$ and σ_ν measures persistence and volatility.

3.2 Domestic firms

Each intermediate goods producing domestic firm j produces a homogenous good according to the CRS production function

$$Y_t(j) = K_{t-1}(j)^\alpha \left[A_t N_t(j)\right]^{1-\alpha}, \tag{8}$$

where A_t is labor-augmenting, aggregate total factor productivity (TFP) and $K_{t-1}(j)$ and $N_t(j)$ denote capital and labor employed by the j-th firm. Productivity grows at a rate $z_t = ln\left(\Gamma_{A,t}\right)$, where $\Gamma_{A,t} = \frac{A_t}{A_{t-1}}$ and z_t follows an AR(1) process

$$z_t = \rho_z z_{t-1} + \varepsilon_{z,t},\ \varepsilon_{z,t} \sim N\left(0, \sigma_z\right), \tag{9}$$

where $-1 < \rho_z < 1$ and σ_z measure persistence and volatility.

With $Y_t \equiv \left(\int_0^1 Y_t(j)^{\frac{\epsilon_t - 1}{\epsilon_t}} dj\right)^{\frac{\epsilon_t}{\epsilon_t - 1}}$ representing the aggregate index for domestic output, firm j faces demand equal to $Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon_t} Y_t^d = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon_t} \left(C_{H,t} + C_{H,t}^f\right)$. The last step assumes that firms operate under producer-currency pricing such that they set the same price in units of the domestic currency irrespective of whether the goods are sold locally or exported. This assumption implies that the law of one price holds at the variety level, i. e. $P_{H,t}(j) = S_t P_{H,t}^f(j)$, where S_t is the bilateral exchange rate.

Firms set prices in a staggered fashion as in Calvo (1983), where $0 < \theta < 1$ is the constant probability that a firm keeps its price fixed in a given period. Following Del Negro and Schorfheide (2013) we depart from Calvo (1983) in assuming that for those firms that cannot adjust prices, $P_{H,t}(j)$ will increase at the geometric weighted average with weights $1 - \iota_P$ and ι_p, respectively, of the steady state inflation, $\bar{\pi}_H$, and of last period's inflation.

Firms that are able to optimally update their price in period t choose the reset price $\widetilde{P}_{H,t}(j)$ to maximize the expected value of current and future profits,

$$\begin{aligned} &\left(\widetilde{P}_{H,t}(j) - (1-\phi)\, mc_t P_{H,t}\right) Y_t(j) + \\ E_t \sum_{s=0}^{\infty} \theta^s \Lambda_{t,t+s} &\left[\widetilde{P}_{H,t}(j)\left(\prod_{l=1}^{s} \pi_{H,t+l-1}^{\iota_P}\bar{\pi}_H^{1-\iota_P}\right) - (1-\phi)\, mc_{t+s} P_{H,t+s}\right] Y_{t+s}(j), \end{aligned} \tag{10}$$

where $\Lambda_{t,t+s} = \beta^s e^{-\frac{g_{t+s}}{g_t}} \left(\frac{C_{t+s}}{C_t}\right)^{-\frac{1}{\tau}} \frac{P_t}{P_{t+s}}$ and ϕ refers to a labor subsidy funded by the domestic government (see Zhang et al. (2021)). The sequence of demand constraints that the re-optimizing firm is facing reads:

$$Y_{t+s}(j) \leq \left(\frac{\widetilde{P}_{H,t}(j)\left(\prod_{l=1}^{s} \pi_{H,t+l-1}^{\iota_P}\bar{\pi}_H^{1-\iota_P}\right)}{P_{H,t+s}}\right)^{-\epsilon_t} \left(C_{H,t+s} + C_{H,t+s}^{f}\right), \tag{11}$$

where $C_{H,t}^{f}$ refers to the foreign consumption of domestic goods. The domestic government sets the labor subsidy such that it compensates the price markup in the steady state, $\mu \equiv (1-\phi)\frac{\epsilon}{\epsilon-1} = 1$. Therefore, the markups can vary over time in the short-run but do not have an effect in the long-run. We define $u_t = ln(\mu_t)$ following an AR(1) process,

$$u_t = \rho_u u_{t-1} + \varepsilon_{u,t},\ \varepsilon_{u,t} \sim N(0,\, \sigma_u), \tag{12}$$

where $-1 < \rho_u < 1$ and σ_u measure persistence and volatility. The law of motion for prices is then given by:

$$P_{H,t} \quad = \quad \left[(1-\theta)\,\widetilde{P}_{H,t}^{1-\epsilon} + \theta\left(\pi_{H,t-1}^{\iota_P}\bar{\pi}_H^{1-\iota_P} P_{H,t-1}\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \tag{13}$$

3.3 Other equilibrium conditions and market clearing

The foreign economy is assumed to be large and an almost closed economy. The foreign country's demand for domestic goods is set up symmetrically to home goods and under the assumption of complete international asset markets, the foreign country has access to the domestic (currency denominated) Arrow-Debreu securities. Together with the definition of the bilateral real exchange rate, $RER_t = \frac{P_t^f S_t}{P_t}$, we can write

$$C_t = \upsilon C_t^f \left(RER_t\right)^{\tau}, \tag{14}$$

where υ is a constant that depends on the initial conditions regarding relative net asset positions. If the law of one price holds under producer-currency pricing, the terms of trade are defined as

$$TOT_t \equiv \frac{P_{F,t}}{S_t P_{H,t}^f} = \frac{P_{F,t}}{P_{H,t}}. \tag{15}$$

Given that the law of one price holds, goods market clearing in the foreign and in the domestic economy yields the risk sharing condition,

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[(1-\gamma)\left(C_t + I_t\right) + \gamma \left(RER_t\right)^{\eta} Y_t^f\right]. \tag{16}$$

Furthermore, the foreign economy inflation, π_t^f, and output, Y_t^f, are assumed to follow AR(1) processes,

$$\pi_t^f = \rho_\pi \pi_{t-1}^f + \varepsilon_{\pi,t},\ \varepsilon_{\pi,t} \sim N\left(0,\, \sigma_\pi\right), \tag{17}$$

respectively

$$Y_t^f = \rho_{Y^f} Y_{t-1}^f + \varepsilon_{Y^f,t},\ \varepsilon_{Y^f,t} \sim N\left(0,\, \sigma_{Y^f}\right). \tag{18}$$

Monetary policy is set according to a standard Taylor-rule specified as

$$\frac{1+i_t}{1+i} = \left(\frac{1+i_{t-1}}{1+i}\right)^{\rho_i} \left[\left(\frac{\pi_t}{\pi}\right)^{\psi_\pi} \left(\frac{Y_t}{Y_t^n}\right)^{\psi_Y}\right]^{1-\rho_i} \varepsilon_{i,t}^{\sigma_i}. \tag{19}$$

The response of the monetary policy interest rate (i. e. the short term interest rate) to inflation and the ouput gap is given by $\psi_\pi > 0$ and $\psi_Y \geq 0$. The inertia parameter ρ_i links the monetary policy rate to its lagged value and $\varepsilon_{i,t}$ is a monetary policy shock with standard deviation σ_i. Inflation is defined as the percent deviation from steady-state inflation and the output gap is defined the percent deviation form the natural output Y_t^n. Since domestic aggregate total factor productivity has a growing trend A_t, the model is stationarized by defining variables in relation to A_t. The full set of the stationarized first order and equilibrium conditions and the equations of the natural output of the economy are listed in appendix A.2.

4 Empirical estimation

In this chapter, we present the econometric methodology and describe the data set. Then, we briefly explain how we allow for economic convergence in the estimation of the model and present the choice of prior assumptions for Bayesian inference.

4.1 Methodology

The Bayesian estimation mainly follows the methodology of Lubik and Schorfheide (2007) for small open economies as in Galí and Monacelli (2005). The vector of observed variables Y_t includes quarterly growth rates in real output, real gross fixed capital formation, the (annualized) inflation rate, the nominal exchange rate and the terms of trade, and the nominal 3-months interbank interest rate and relates to the model variables as follows:

$$Y_t = \begin{bmatrix} \frac{\tau+\tau\varphi}{1+\tau\varphi}\Gamma_Q + \left(\Delta\frac{Y_{A,\,t}}{Y_A^{ss}} + \frac{\tau+\tau\varphi}{1+\tau\varphi}\frac{A_t}{A^{ss}}\right) * 100, \\ \frac{\tau+\tau\varphi}{1+\tau\varphi}\Gamma_Q + \left(\Delta\frac{I_{A,\,t}}{I_A^{ss}} + \frac{\tau+\tau\varphi}{1+\tau\varphi}\frac{A_t}{A^{ss}}\right) * 100, \\ \pi_A + ln\left(\frac{\pi_t}{\pi^{ss}}\right) * 400, \\ ConsNER + \left(\Delta\frac{RER_t}{RER^{ss}} + ln\frac{\pi_t}{\pi^{ss}} - ln\frac{\pi_t^f}{1}\right) * 100, \\ ConsTOT - (TOT_t - TOT_{t-1}) * 100, \\ \pi_A + r_A + \frac{\tau+\tau\varphi}{1+\tau\varphi}\Gamma_Q * 4 + ln\left(\frac{i_t}{i^{ss}}\right) * 400 \end{bmatrix}$$

The observables enter the model in levels, i. e. the observed variables are not demeaned as in Lubik and Schorfheide (2007). Recall that the model variables enter in their stationarized form, i. e. adjusted by productivity A_t. Hence, the observed output growth corresponds to the sum of output growth and productivity growth, Γ_Q. Γ_Q is estimated and linked to the structural parameters of the model. Further steady state-related parameters, which have to be estimated, are π_A, $ConsNER$, $ConsTOT$ and r_A relating to the inflation rate, the nominal exchange rate, the terms of trade and the interest rate.[4] Therefore, the model takes into account non-zero growth and inflation in the steady state and the discount factor, β is estimated as well. The likelihood function is constructed using the Kalman filter based on the state-space representation of the model. Through the likelihood function the data is used to update a prior distribution (section 4.3). Markov Chain Monte Carlo (MCMC) simulation is used to generate parameter draws from the posterior distributions. The posterior mode, obtained by maximizing the log posterior kernel, is chosen as the starting point for the the Metropolis-Hastings algorithm sampling the posterior distribution of the parameters. The number of replications of the Metropolis-Hastings sampler is set to 250,000 and the number of parallel chains for the Metropolis-Hastings algorithm is set at 2. Drawing the proposal from a jumping distribution, the scale factor that multiplies the inverse of the Hessian matrix that is computed at the posterior mode (i. e. the variance) is chosen as 0.35, which leads to an acceptance rate between around 0.2 to 0.25.

4.2 Data description

The sample covers 68 observations at quarterly frequency and spans the period between Q1 2003 and Q4 2019. Real gross domestic product (output) and real gross fixed capital formation (investment) are calendar and seasonally adjusted (chained, 2010 reference year). Inflation rates are based on the seasonally-adjusted Harmonized Index of Consumer Prices (HICP) at constant taxes (all-items) and defined as log differences multiplied by 400 to obtain annualized percentage changes. Since the Euro Area is the main trading partner of the four countries, nominal exchange rates are expressed in local currency per Euro and extracted from Macrobond Financial AB. The terms of trade are calculated by dividing the calendar and seasonally adjusted price index of imports of goods and services by the price index of exports of goods and services; both in local currencies. Percentage changes in output, investment, the terms of trade, and the nominal exchange rates are obtained by multipliying the quarterly log differences by 100. The main monetary policy interest rates are approximated by 3-month interbank interest rates and extracted from Refinitiv Eikon (expect for the Czech 3-month interbank rate (PRIBOR), which is downladed directly from

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[4] Here, Δ is defined as the log-dfference.

the Czech National Bank). Note that the time series are not demeaned. Except for the 3-month interbank interest rates, and nominal exchange rates, the data source is Eurostat (ESA 2010) and, therefore, consistent across the four EU member countries. While r^*-estimates for advanced economies often cover considerably longer periods (for example, the time series of Holston et al. (2017) start in 1961), the four countries officially adopted inflation-targeting regimes in 1998 (Czech Republic), 1999 (Poland), 2001 (Hungary) and 2005 (Romania), partly following periods of stark disinflation after the fall of the iron curtain in 1989 and preparing for European Union membership in 2004 (Poland, Czech Republic, Hungary) and 2007 (Romania). The data description is summarized in Table 9 in the appendix.

4.3 Choice of priors

The prior distributions are mostly chosen in accordance with Lubik and Schorfheide (2007) and Zhang et al. (2021) and summarized in table 2. Basically, prior densities, means and standard deviations are set identical for all four countries, although some country-specific differences arose during the estimation process, which are discussed below (section 5.1). For the inverse Frisch elasticity of labor supply and the elasticity of substitution between home and foreign goods we choose Gamma distributions with prior means of 1.5 respectively 1.0, as in Zhang et al. (2021). The priors of the parameters of the monetary policy rule are centered around values, which are typically associated with standard Taylor rules (0.5 and 1.5 for ψ_x respectively ψ_π). The smoothing parameter of the monetary policy rule, ρ_i, has a prior mean of 0.5.

The prior assumptions of the parameter of trade openness, γ, deserve some attention, since it seems that there is no consensus in the literature. Lubik and Schorfheide (2007) use a prior tightly centered at 0.2, while Zhang et al. (2021) take the import shares, calculated as the average of imports of goods and services to GDP over the sample period. In our case, imports of goods and services to GDP between 2003 and 2019 averaged 43 % in case of Poland, 65 % for the Czech Republic, 75 % for Hungary and 39 % for Romania (source: World Development Indicators, World Bank). Note that these shares are much higher than the import shares of the (advanced) economies in Zhang et al. (2021), which average

0.24 across six developed economies. In CEE, firms are typically part of international supply chains, for example in the automotive sector. Intermediate goods are imported for production (assembling) and exported for final consumption abroad. However, in the model, the parameter γ is the share of foreign goods imported for domestic consumption (see equation (2)), which in our cases can differ significantly from the import share. McKnight et al. (2020) proxy the trade openness by the ratio of the average share of real imports in GDP to the average share of real consumptions to GDP. If we used this approximation, the prior means would have to be set even higher. Alternatively, if we crosscheck with data from the last World Input-Output Database (WIOD, 2016 release) of the University of Groningen, the share of imports from all countries for final use for household consumption of total household consumption is 0.17 in case of Poland, 0.21 for Czech Republic, 0.22 for Hungary and 0.14 for Romania. In the base case, we set the mean prior of γ at 0.30 respectively 0.40; see below for further discussion.

The elasticity of intertemporal substitution, τ, has a Beta prior, which is centered at 0.5. The Calvo parameter for price stickiness is also assumed to follow a Beta distribution and has a prior mean of 0.66. The assumption of a normal prior with mean of 0.3 is applied for the factor share, α. Investment adjustment cost has a normal prior with mean 4.0 (see Smets and Wouters (2003)). For the priors of the exogenous shock processes, Lubik and Schorfheide (2007) conduct a pre-sample analysis for world output, inflation and domestic technology and we follow their prior specifications of the persistence and standard deviation parameters. The specifications of persistence and standard deviation parameters of the domestic preference shock and the domestic cost-push shock are set as in Zhang et al. (2021). Persistence and standard deviation of the investment shock is assumed to follow a Beta distribution with prior mean of 0.80 and an inverse Gamma prior with a mean of 0.25.

4.4 Convergence

As emphasized, our four example economies we estimate the model on have experienced a process of rapid convergence over our estimation horizon. Their experience is thus markedly different than for a typical advanced economy, on which model estimation is typically

Parameter	Description	Range	Density	Mean	Std. Dev.
φ	inverse Frisch	(0,10]	Gamma	1.50	0.50
η	elasticity H-F goods	(0,10]	Gamma	1.00	0.20
τ	intertemporal ES	(0,1)	Beta	0.50	0.20
γ	trade openness	[0,1)	Beta	0.30 (0.40)	0.05
θ	Calvo parameter	(0,1]	Beta	0.66	0.20
α	factor shares	[0,1)	Normal	0.30	0.05
κ_I	investment adjustment cost	[2,20]	Normal	4.00	1.50
δ	depreciation rate	(0,0.05]	Gamma	0.025	0.010
ψ_Y	monetary rule, output	(0,5]	Gamma	0.50	0.25
ψ_π	monetary rule, inflation	(1,10]	Gamma	1.50	0.50
ρ_i	monetary rule, smoothing	(0,1)	Beta	0.50	0.20
r_A	interest rate level	(0,2]	Gamma	1.00	0.20
π_A	inflation level	(0,12]	Gamma	4.00	20
Γ_Q	output level	(0,2]	Normal	0.75	0.20
$consTOT$	ToT level	[-2,2]	Normal	0.0	0.10
$consNER$	exchange rate level	[-2,2]	Normal	0.0	0.10
ι_P	inflation indexation	(0,1]	Beta	0.50	0.10
ρ_a	technology persistence	(0,1)	Beta	0.20	0.10
ρ_{y^f}	world output persistence	(0,1)	Beta	0.90	0.05
ρ_{π^f}	world inflation persistence	(0,1)	Beta	0.80	0.10
ρ_g	preference persistence	(0,1)	Beta	0.20	0.10
ρ_u	markup persistence	(0,1)	Beta	0.80	0.10
ρ_ν	investment persistence	(0,1)	Beta	0.80	0.10
σ_i	std. dev. MP rule	(0,10]	InvGamma	0.50	0.20
σ_q	std. dev. terms of trade	(0,10]	InvGamma	1.50	0.55
σ_z	std. dev. technology	(0,10]	InvGamma	1.00	0.35
σ_{y^f}	std. dev. world output	(0,10]	InvGamma	1.50	0.35
σ_{π^f}	std. dev. world inflation	(0,10]	InvGamma	0.55	0.20
σ_g	std. dev. preference	(0,10]	InvGamma	0.25	0.10
σ_u	std. dev. markup	(0,10]	InvGamma	0.55	0.20
σ_ν	std. dev. investment	(0,1]	InvGamma	0.25	0.10

Table 2: Prior distributions of the parameters.

performed. Our model with capital allows us to explicitly capture the idea that part of the behavior of our observable macroeconomic time series of our four CEE countries is driven by a process of capital accumulation, via which these economies are catching up to their respective long run growth path, i.e. convergence taking place. In technical terms, we proceed as follows. In the estimation process, the DSGE model solution – for each parameter draw – is mapped into the format of the Kalman filter, which is a linear filter consisting of a transition equation for observables and and an equation for transition of (latent) state variables. The state vector is equal to the vector of state variables of the DSGE model itself, both exogenous (shock processes) and endogenous, which, importantly, includes the capital stock. Variables are expressed in terms of (percentage) deviations from their respective steady state.

A common choice in estimation of macroeconomic models is to initialize the state vector of the Kalman filter at the stationary distribution, i.e. to choose an initial state vector of zeros (state variables being zero percent away from their steady state) and setting the initial matrix of the variance of the error of forecast equal to the unconditional variance of the state variables. This logic makes sense for advanced economies, for which case fluctuations in their time series largely reflect fluctuations around their growth path. For our CEE economies, however, fluctuations in the macroeconomic time series of these countries, to a large degree reflect the process of capital accumulation. To account for this, we initialize the starting value of the state vector pertaining to the capital stock to below steady state, while currently keeping the initialization of the error of forecast at the unconditional variance of the state variables as well.[5]

Since we assume that the capital stocks are not observed, we make use of the empirical beta estimates of convergence (section 2) to help us identify the approximate initial level of the capital stocks. According to our regressions, if we assume a $beta = 0.02$, i.e. annual catch-up of 2 %, for a period of 17 years between 2003 and 2019, the initial level of the capital stocks is assumed approximately 29 % below the steady state level, i. e. $K_{i,Q1\,2003} =$

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[5] Alternatively, we could consider setting up the initial state covariance matrix to be a diagonal matrix where each diagonal element is set to a very large number (i.e., adopting Dynare's diffuse Kalman Filter). This choice would reflect our knowledge that the initial state of the capital stock is certainly below steady state, but leaves room for uncertainty in initialization, as precisely where the initial state of the model lies is not entirely known.

$0.71 * K_{i,ss}$, where $K_{i,ss}$ is the steady state capital of country i. In our baseline approach to initialize capital stocks, we therefore do not allow for differences of our four CEE countries; however, in section 5.2.1 we explore an alternative approach to find guidance for initial levels of the capital stock of the four economies individually.

5 Results

5.1 Baseline estimation

In the section, the estimation results of the small open economy model are presented. Table 3 shows the posterior means and 90 % posterior probaility intervals of the structural, policy and steady state-related model parameters. Table 4 shows the posterior means of the persistence parameters and standard devations of the shock processes. The parameters of the Taylor rule indicate that central banks in CEE respond strongly to changes of inflation with posterior estimates of ψ_π far beyond the prior assumption (2.6 versus 1.5), while the response to changes in the output gap appears benign. The anti-inflationary bias may be due to previously high inflation during the transition from former socialist regimes to market economies during the 1990s and the preparation for accession, respectively, membership of the EU. The mean posterior estimates for interest rate smoothing, ρ_i, average 0.79 across the four economies and are higher than the prior assumption and also higher than estimates of Caraiani (2013) for Poland, Czech Republic and Hungary (for a Taylor rule that additionally includes an exchange rate term). The estimates of the posterior mean of the intertemporal elasticity of substitution are on average above the prior means (0.69 versus 0.50) but still in line though on the higher end compared to the empirical evidence (Havránek, 2015). As already discussed (section 4.3), there is some ambiguity about the choice of the prior assumptions for the trade openness parameter, γ. Eventually, we set the prior mean to 0.4 (Poland, Czech Republic) respectively 0.3 (Hungary, Romania). In this order, the posterior mean estimates are 0.39, 0.35, 0.29 and 0.16. The implied Frisch elasticities come close the prior mean of 1.50, except of the posterior mean in case of Poland (0.66). The posteriors means of the elasticity of substitution between home and foreign goods range between 0.72 (Poland) and 1.45 (Czech Republic). The posterior means of the parameter of price stickiness are lower than the prior mean of 0.66, which might be due to past high inflation and to periods of rapid price adjustment during and after the transition to market economies. Three parameters are connected to the estimation of capital: capital adjustment costs, the depreciation rate and the factor share. The posterior means of the capital adjustement cost parameter, κ_I, are broadly as expected (on average 4.31 versus a prior mean of 4.00). The posterior mean estimates of the depreciation rates are also close to their prior mean (0.03). To simulate the convergence through capital accumulation, the initial values of the capital stocks are set well below the steady states (see section 4.4), therefore the posterior means of the factor share, α, which have a mean of 0.19, fall short of the assumption of a capital share of 1/3 (usually made for advanced economies). To account for this, prior assumption of the lower bound was decreased from 0.2 to 0.1 (except for Poland), after the estimation failed to converge. Posterior estimates of the persistence parameters of the shocks are broadly in line with the prior assumption (only persistence of the technology process appears low by averaging 0.08). The volatility of the posterior means of the investment shock, σ_ν, becomes quite high, which is probably not unusual. For example, Justiniano et al. (2011) estimate a posterior median of 5.8 for the U. S.[6]

Next, we are going to establish the main results concerning the natural rate of interest in small open economies in CEE. We expect that for CEE economies the natural rate is higher compared to estimates for advanced economies. Convergence drives down the real interest rate. Moreover, the drivers of a decline that were found for advanced economies are also prevailing in CEE, such as demographic change, the shortage of safe assets or secular stagnation (as described in section 1). Hence, we expect a downward trend in r^*. Figure 2 shows the estimates from the baseline estimation. R^* has been declining in all four economies since the beginning of the sample period in 2003. This is consistent with the consensus in the literature about a r^*-decline, as outlined in section 1. On average between the years 2003 and 2019, we estimate a natural rate of interest of 2.0 % for Poland, 0.4 % for the Czech Republic, 2.7 % for Hungary and 3 % for Romania. The magnitudes appear consistent with estimates of TFP growth during 2003 to 2019 (AMECO). Consistent with

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[6] The high standard deviations of σ_ν are likely linked to the structure that we impose on capital formation with inital capital levels far away from steady states and with the observed gross fixed investment time series. For example, one could possibly adress this by introducing capital utilization to the model.

Parameter	Prior mean	Post. mean	90 % interval	Prior mean	Post. mean	90 % interval
	Poland			Czech Republic		
τ	0.50	0.68	[0.49,0.89]	0.50	0.66	[0.45,0.89]
γ	0.40	0.39	[0.29,0.48]	0.40	0.35	[0.27,0.43]
η	1.00	0.72	[0.5,0.93]	1.00	1.45	[1.11,1.77]
φ	1.50	0.66	[0.25,1.07]	1.50	1.12	[0.55,1.67]
θ	0.66	0.09	[0.02,0.15]	0.66	0.20	[0.07,0.32]
κ_I	4.00	3.88	[2.85,4.92]	4.00	6.07	[4.78,7.36]
δ	0.03	0.03	[0.02,0.04]	0.03	0.04	[0.03,0.05]
α	0.30	0.22	[0.2,0.23]	0.30	0.18	[0.14,0.21]
ψ_Y	0.50	0.10	[0.03,0.17]	0.50	0.06	[0.02,0.11]
ψ_π	1.50	3.08	[2.22,3.87]	1.50	2.77	[1.96,3.54]
ρ_i	0.50	0.83	[0.77,0.89]	0.50	0.84	[0.78,0.9]
r_A	1.00	0.79	[0.54,1.04]	1.00	0.69	[0.46,0.91]
π_A	4.00	2.08	[1.17,3]	4.00	1.14	[0.57,1.72]
Γ_Q	0.75	0.42	[0.24,0.6]	0.75	0.09	[0,0.17]
$consTOT$	0.00	-0.06	[-0.22,0.11]	0.00	-0.03	[-0.12,0.06]
$consNER$	0.00	0.01	[-0.16,0.17]	0.00	-0.01	[-0.18,0.15]
ι_P	0.50	0.48	[0.32,0.65]	0.50	0.47	[0.3,0.64]
	Hungary			Romania		
τ	0.50	0.73	[0.55,0.92]	0.50	0.68	[0.44,0.93]
γ	0.30	0.29	[0.22,0.37]	0.30	0.16	[0.1,0.23]
η	1.00	1.28	[0.94,1.64]	1.00	1.01	[0.68,1.36]
φ	1.50	1.14	[0.57,1.72]	1.50	1.35	[0.64,2.01]
θ	0.66	0.42	[0.3,0.54]	0.66	0.46	[0.08,0.79]
κ_I	4.00	4.10	[2.83,5.33]	4.00	3.21	[2.19,4.21]
δ	0.03	0.04	[0.03,0.05]	0.03	0.02	[0.01,0.03]
α	0.30	0.17	[0.14,0.2]	0.30	0.20	[0.16,0.24]
ψ_Y	0.50	0.24	[0.1,0.37]	0.50	0.24	[0.08,0.37]
ψ_π	1.50	2.22	[1.47,2.96]	1.50	2.50	[1.55,3.45]
ρ_i	0.50	0.79	[0.72,0.87]	0.50	0.71	[0.61,0.81]
r_A	1.00	0.89	[0.6,1.17]	1.00	0.92	[0.63,1.22]
π_A	4.00	2.24	[0.88,3.52]	4.00	2.71	[1.16,4.29]
Γ_Q	0.75	0.37	[0.21,0.53]	0.75	0.56	[0.33,0.78]
$consTOT$	0.00	-0.06	[-0.17,0.05]	0.00	0.15	[-0.01,0.3]
$consNER$	0.00	0.00	[-0.16,0.16]	0.00	0.02	[-0.15,0.18]
ι_P	0.50	0.44	[0.28,0.61]	0.50	0.42	[0.24,0.6]

Table 3: Posterior estimation results of the model with convergence: Posterior mean, lower and upper bound of 90 % HPD interval.

Parameter	Prior mean	Post. mean	90 % interval	Prior mean	Post. mean	90 % interval
		Poland			Czech Republic	
ρ_a	0.20	0.06	[0.01,0.1]	0.20	0.04	[0.01,0.08]
ρ_{y^f}	0.90	0.98	[0.96,0.99]	0.90	0.96	[0.93,0.98]
ρ_{π^f}	0.80	0.49	[0.35,0.64]	0.80	0.54	[0.39,0.69]
ρ_g	0.20	0.25	[0.06,0.43]	0.20	0.26	[0.06,0.45]
ρ_u	0.80	0.87	[0.79,0.96]	0.80	0.79	[0.69,0.89]
ρ_ν	0.80	0.69	[0.54,0.83]	0.80	0.81	[0.71,0.91]
σ_i	0.50	0.25	[0.19,0.32]	0.50	0.21	[0.16,0.25]
σ_a	1.00	1.68	[1.14,2.2]	1.00	1.60	[1.2,2.01]
σ_{y^f}	1.50	2.05	[1.48,2.63]	1.50	1.12	[0.89,1.37]
σ_{π^f}	0.55	3.89	[3.32,4.47]	0.55	2.29	[1.94,2.64]
σ_g	0.25	0.18	[0.12,0.23]	0.25	0.16	[0.11,0.2]
σ_u	0.55	2.17	[1.29,3.03]	0.55	1.55	[0.95,2.11]
σ_q	1.00	0.45	[0.31,0.59]	1.00	0.41	[0.31,0.52]
σ_ν	0.25	8.53	[7.06,10]	0.25	4.24	[3.17,5.29]
		Hungary			Romania	
ρ_a	0.20	0.07	[0.02,0.11]	0.20	0.14	[0.01,0.32]
ρ_{y^f}	0.90	0.91	[0.87,0.96]	0.90	0.95	[0.92,0.99]
ρ_{π^f}	0.80	0.46	[0.32,0.62]	0.80	0.66	[0.49,0.84]
ρ_g	0.20	0.23	[0.05,0.4]	0.20	0.42	[0.09,0.83]
ρ_u	0.80	0.92	[0.86,0.98]	0.80	0.74	[0.56,0.94]
ρ_ν	0.80	0.50	[0.35,0.63]	0.80	0.57	[0.42,0.72]
σ_i	0.50	0.42	[0.31,0.51]	0.50	0.60	[0.45,0.75]
σ_a	1.00	1.50	[1.16,1.85]	1.00	2.30	[1.51,3.13]
σ_{y^f}	1.50	1.11	[0.9,1.32]	1.50	1.54	[1.15,1.93]
σ_{π^f}	0.55	3.04	[2.58,3.49]	0.55	1.53	[0.94,2.07]
σ_g	0.25	0.20	[0.12,0.27]	0.25	1.05	[0.12,2.49]
σ_u	0.55	1.73	[1.05,2.41]	0.55	2.84	[0.3,5.12]
σ_q	1.00	0.45	[0.33,0.56]	1.00	1.78	[1.36,2.22]
σ_ν	0.25	15.22	[11.55,20]	0.25	16.82	[13.55,20]

Table 4: Posterior estimation results of the model with convergence: Shock parameters. Posterior mean, lower and upper bound of 90 % HPD interval.

our r^*-estimates, TFP growth was highest for Romania with 2.2 % and second-highest for Poland with 1.5 %. For Czech Republic average TFP growth was also 1.5 %, while the average r^* was lower. This could be due to a safe asset shortage, respectively the low supply of goverment bonds of a small country with low public debt. In case of Hungary the average TFP growth was low (0.4 % according to AMECO). R^* turned out statistically significantly negative in the Czech Republic in 2012 and remained negative until mid-2018, before we see a pick-up. Hledik and Vlček (2018) estimate a natural interest rate for the Czech Republic, that neither has a sustained downward trend but has been hoovering around 1 % since 2000 and never dropped into negative territory, while according to our results, r^* was negative for several years. An increase in r^* is also visible in case of Hungary towards the end of the etimation period. There is a considerable drop mostly in the aftermath of the Global Finanical Crisis (GFC), so we take the GFC as a cut-off point (Q3 2008) for comparison. Then, before the GFC r^* averaged 2.7 %, 1.0 %, 3.7 % and 6.0 % in Poland, Czech Republic, Hungary and Romania, while during the last five years of the total period (after Q4 2014) it averaged 1.0 %, 0.0 %, 1.2 % and -0.1 %.

Further main results concern the evolution of the capital stocks shown in figure 3. The empirical identification of the initial levels of the capital stock using evidence described in section 2 (29 % below the steady state) consistently leads to posterior estimates of the capital stocks at the start of the estimation period near the values of initialization. All countries have in common a rather steady convergence path, although plausible differences can be observed. A rapid capital accumulation sets in after the GFC and leads to capital stocks near their steady states towards the ends of the estimation period. Poland's initial capital stock starts closest to its steady state level among the four countries. Given Poland maintained positive growth through the GFC as well as during the euro crisis without falling into a recession, it shows a steady convergence of the capital stocks that even goes beyond the steady state. This might also be driven by the large capital inflow from the EU's structural and cohesion funds, which additionally has been fuelling capital accumulation. In 2003, living standards were highest in the Czech Republic (see figure 1), hence, also the inital capital stock is similarly close to the steady state level but does not go beyond the steady state (as in the case of Poland). For Romania, the trajectory of the posterior

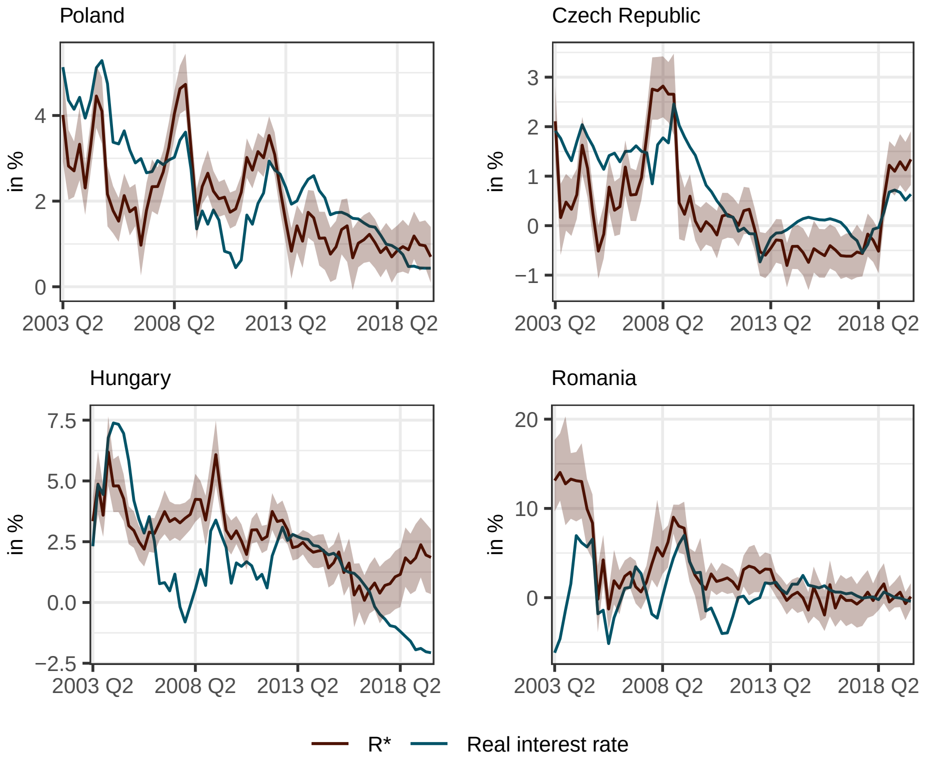

Figure 2: Natural rates of interest in converging economies. The graph shows r^* measured as the (annualized) median posterior estimates. Real short-term interest rates are calculated as the 3-months interbank interest rates minus survey inflation expectations. Inflation expectations are measured based on the European Commission's Business and Consumer Survery following the Carlson and Parkin (1975) method. HPD interval is the upper/lower bound of the 90 % highest posterior density interval.

estimates falls below the initialization and indicates a capital stock furthest away from the steady state, which is plausible since Romania started from the lowest development stage in 2003.

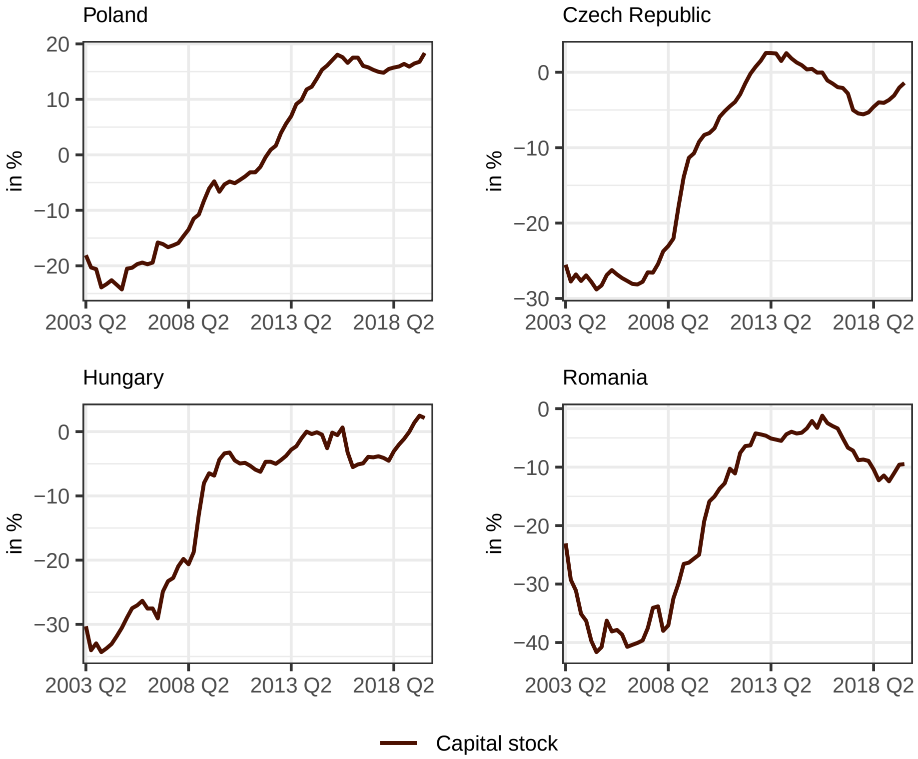

Figure 3: Median posterior estimates of capital stocks in converging economies

5.2 Alternative estimations

The following section compares the estimation results of the baseline model to alternative model estimates. We consider a model specification with different inital levels of individual capital stocks and a model without capital (labor only).

5.2.1 Alternative initial capital stock

Alternatively, we calculate the GDP p. c. of the emerging economy in relation to Germany's GDP p. c. in the years 2003 and 2019. For example, Poland's GDP p. c. was 0.40 and

0.62 of Germany's GDP p. c. in 2003 respectively 2019.[7] Hence, the catch-up was 0.22 and the initial capital stock is set at $0.78 * K_{i,ss}$. The other economies caught up from 0.62 to 0.76 (Czech Republic), from 0.51 to 0.61 (Hungary) and from 0.33 to 0.56 (Romania) of Germany's GDP p. c. The initial capital stocks are set accordingly at 0.87, 0.90 and 0.77 of the steady state value of capital.[8] The posterior estmation results of the model parameters with alternative initial values of the capital stocks are similar to the baseline estimation (appendix A.4, tables 10 and 11). Figure 4 depicts the median posterior estimates for r^*. The trajectories are similar to the r^*-estimates of the baseline estimation. Differences arise with respect to the evolution of the capital stocks; shown in figure 5. Capital accumulation starts closer to the steady states though the trajectories appear similar to the capital stocks of the baseline estimation.

5.2.2 Model without capital

Alternatively, a version of the small open economy model without capital (labor only) is estimated; compare Zhang et al. (2021).[9] Table 12 in appendix A.4 reports detailed Bayesian estimation results for the parameters of the model. Overall, the posterior estimates are comparable to our baseline model, but some differences arise. Concerning the monetary policy rule, central banks react with sufficient strength to inflation changes with mean posterior estimates of the parameter ψ_π ranging between 1.16 (Czech Republic) and 2.54 (Hungary), alhough in the baseline model the response is even stronger (between 2.22 for Hungary and 3.08 for Poland), in our view, reflecting better the strong anti-inflationary

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[7] The data source is the IMF World Economic Outlook, April 2021, GDP per capita at purchasing power parities.

[8] The initial level of the capital stock of country i is given by

$$K_{i,Q1\,2003} = K_{i,ss} * \left[1 - \left[\frac{GDPpc_{i,2019}}{GDPpc_{DE,2019}} - \frac{GDPpc_{i,2003}}{GDPpc_{DE,2003}}\right]\right].$$

[9] A model appendix of the version with labor only in the production function is available upon request. The model described in section 3 but without capital in the production function (equation 8), is essentially derived in Zhang et al. (2021). Some differences arise: We estimate the trend growth rate (Γ_Q), so that it is updated and linked to the structural parameters. The same holds for the trend inflation rate, the interest rate, the terms of trade and the nominal exchange rate. Furthermore, we solve endogenously for the terms of trade, while they are exogenous in Zhang et al. (2021) thereby following Lubik and Schorfheide (2007). In our case, the observables are not demeaned, hence, the translation to the model variables is different (section 4.1).

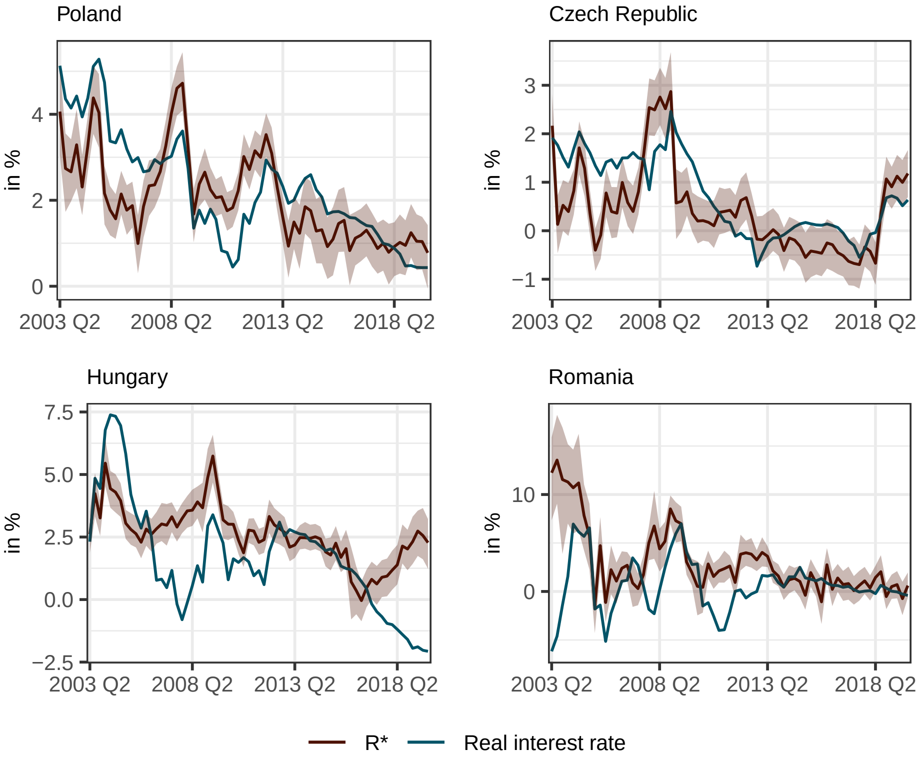

Figure 4: Natural Rates of Interest in Small Open Economies (with alternative initial capital stocks). The graph shows the natural rates of interest measured as the (annualized) mean posterior estimates. Real short-term interest rates are calculated as the 3-months interbank interest rates minus survey inflation expectations. Inflation expectations are measured based on the European Commission's Business and Consumer Survery following the Carlson and Parkin (1975) method. HPD interval is the upper/lower bound of the 90 % highest posterior density interval.

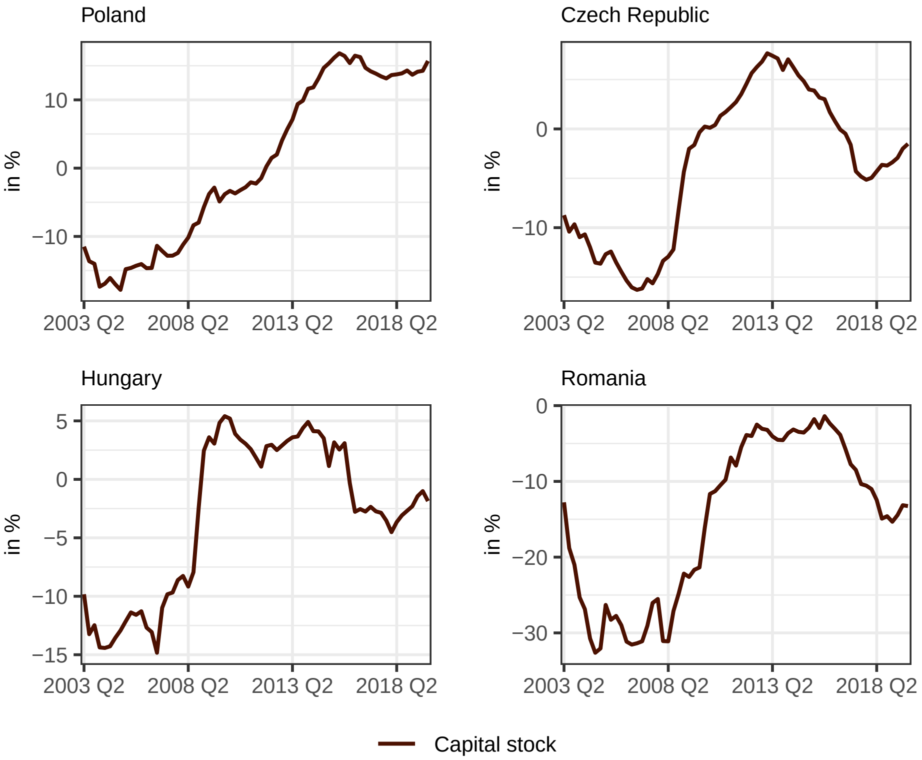

Figure 5: Median posterior estimates of capital stocks with alternative initial values.

strategy during that period (see section 5.1). The response to changes in output is again benign. There is considerable interest rate smoothing, as is the case in the baseline model. The posterior means of the trade openness parameter, γ, appear on the lower end of the spectrum with 0.08 (Poland), 0.18 (Czech Republic), 0.25 (Hungary), and 0.18 (Romania). The higher posterior estimates of the baseline model (0.39, 0.35, 029 and 0.16) seem more appropriate given the significant openness to trade of the four economies (as already discussed). The posterior means of the intertemporal elasticity of substitution are comparably high with estimates averaging 0.89 across the four countries, and our baseline estimation appears better in line with previous findings (Havránek, 2015). Our posterior means of the inverse Frisch elasticity of labor supply, φ, estimated between 1.20 (Poland) and 1.69 (Hungary), imply Frisch elasticities between 0.59 and 0.83. The posteriors means of the elasticity of substitution between home and foreign goods are between 0.80 (Poland) and 1.06 (Hungary). Table 13 in appendix A.4 lists the posterior estimates of the persistence parameters and standard deviations of the stochastic processes of the exogenous shocks. The estimates of the persistence parameters of the shock processes are basically in line with

	PL	CZ	HU	RO
Baseline model:				
$mean_{2003Q2-2019Q4}$	2.0	0.4	2.7	3.0
$mean_{2003Q2-2008Q3}$	2.7	1.0	3.7	6.0
$mean_{2014Q4-2019Q4}$	1.0	0.0	1.2	-0.1
$mean_{2014Q4-2019Q4} - mean_{2003Q2-2008Q3}$	-1.7	-1.0	-2.4	-6.1
Model with alternative initial capital:				
$mean_{2003Q2-2019Q4}$	2.1	0.4	2.6	3.0
$mean_{2003Q2-2008Q3}$	2.6	1.0	3.3	5.4
$mean_{2014Q4-2019Q4}$	1.1	-0.1	1.5	0.7
$mean_{2014Q4-2019Q4} - mean_{2003Q2-2008Q3}$	-1.6	-1.0	-1.9	-4.7
Model without capital:				
$mean_{2003Q2-2019Q4}$	2.2	1.0	1.9	2.1
$mean_{2003Q2-2008Q3}$	3.1	1.4	4.7	6.2
$mean_{2014Q4-2019Q4}$	1.2	0.8	-1.7	-2.0
$mean_{2014Q4-2019Q4} - mean_{2003Q2-2008Q3}$	-1.9	-0.6	-6.4	-8.2

Table 5: R^* Summary. Historical means of different median posterior estimates of r^*.

the literature (Zhang et al., 2021) and the persistence found in the data, except for low values of the technology shock (similar to the baseline model). Implications for monetary policy of different models are discussed in the next section.

5.3 Discussion

Taking stock of the presented results we find significant implications for monetary policy. Table 5 summarizes the median posterior r^*-estimates of the different models. There is a broad consensus between the two model specifications with capital accumulation and convergence about the trajectory, however, some differences emerge when we compare them to the model without capital. Especially, for Hungary and Romania the median estimates fall more into negative territory than in the two models with capital. On average, r^* falls to -1.7 % respectively -2.0 % between Q3 2014 and Q4 2019 in the two countries.

Figure 6 depicts the gap between the actual real interest rate and the real natural rate of interest, which acts as a guide to monetary policy. In times the actual real interest rate is higher than r^*, monetary policy is contractionary, whereas when the actual real interest rate is lower than r^*, the monetary policy stance is expansionary. The interest rate gap is shown for both the baseline model and the model without capital. Until around 2008, both estimates of r^* signal that monetary policy was too tight in Poland and Czech Republic. For Hungary, only r^* of the model with capital indicates a tight monetary policy until around 2006, and too expansionary afterwards as indicated by both models until around 2013. In Romania, monetary policy appears to loose until around 2013, afterwards, both r^*-estimates indicate a restrictive stance. After the GFC, we find some periods of conflicting signals from different models. For Hungary, the differing r^*-trajectories send ambiguous signals for monetary policy after around 2013. Also in the case the Czech Republic, we find some disagreement between the two interest rate gaps after the GFC. After around 2010, the model with capital indicates that monetary policy was too contractionary, while the model version without capital indicates that monetary policy was too expansionary after 2013. Starting in 2018, both estimates suggest that monetary policy should have been gradually more restrictive.

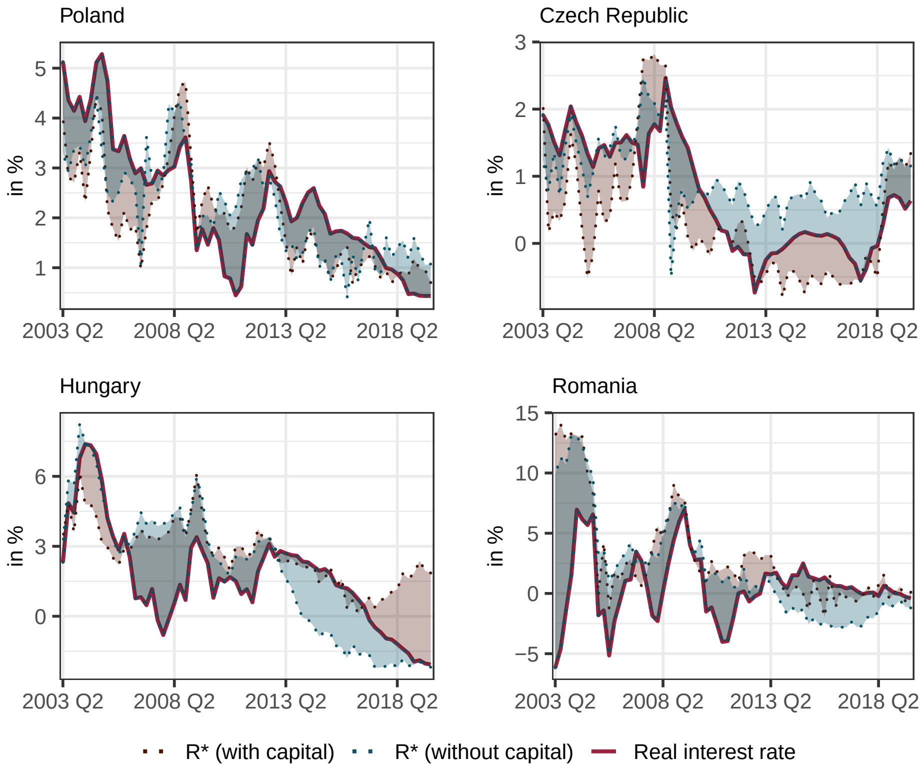

Figure 6: Natural Rates of Interest in Small Open Economies. The graph shows the real interest rate gaps for the models with (red) and without capital (blue green). Shaded areas show the gaps between actual real short-term interest rates and median posterior estimates of r^*.

Finally, the framework allows us to gain insights about the relationships between the natural rate of interest, the rental rate of capital and the return on capital. It is a well known fact (Gomme et al., 2011, 2015; Caballero et al., 2017; Marx et al., 2021; Reis,

2022) that measures of the return on capital have been remarkably stable over the last decades. The deviation between the dramatic decline in the natural rates of interest and the stable (measured or observed) rates of return on capital is an unsolved puzzle in the literature. This literature focuses mostly on advanced economies, foremost the U.S. We present measures of the real return on capital based on time series from the European Commission's AMECO database in figure 7, documenting that in our CEE countries we similarly did not observe a decline compared to the paths of real natural rates, but instead they are rather stable. Gomme et al. (2011) discuss constructing returns on capital from national accounts. Here, the observed (average) return on capital is constructed by dividing the series 'Net operating surplus, total economy' by the 'Net capital stock at constant prices; total economy', which is multiplied by the 'Price deflator gross fixed capital formation: total economy' from the AMECO database (for example, as in Broner et al. (2021)). To evaluate our model implications on these time series, we juxtapose the posterior median estimates of the rental rates of capital and of the expected returns on capital with measures of the real return on capital based on the time series contructed from AMECO data in figure 7. The figure suggests that these time series share similar dynamics. By eyeballing, the estimated rental rate of capital, r_t^K, resembles approximately the observed returns on capital with regards to size and shape. In the case of Poland and Romania, the observed rates of return lie within the 90 % credible intervals of r_t^K, although our model framework does not take into account the riskiness of capital investment. We also estimate the expected return on capital, given by the term in squared brackets of equation 27, $r_{t+1}^K + q_{t+1}(1-\delta)$, divided by marginal value of installed capital, q_t, respectively its shadow price or Tobin's q. However, in this case, the similarity to the AMECO based time series vanishes and the DSGE-model implied estimated expected rate of return on capital resembles closely the path of natural rates of interest. Table 6 provides a summary. When we compare again the periods before 2008 and after 2014, on average across the four economies the declines in r^* and the estimated returns on capital are around 83 % and 99 %. Then again, the declines of the estimated rental rates of capital are much lower averaging about 15 % and the observed returns on capital decrease by about 3 %. So, in our framework, the rental rates of capital come close to empirically observed returns on capital, while estimated returns on capital

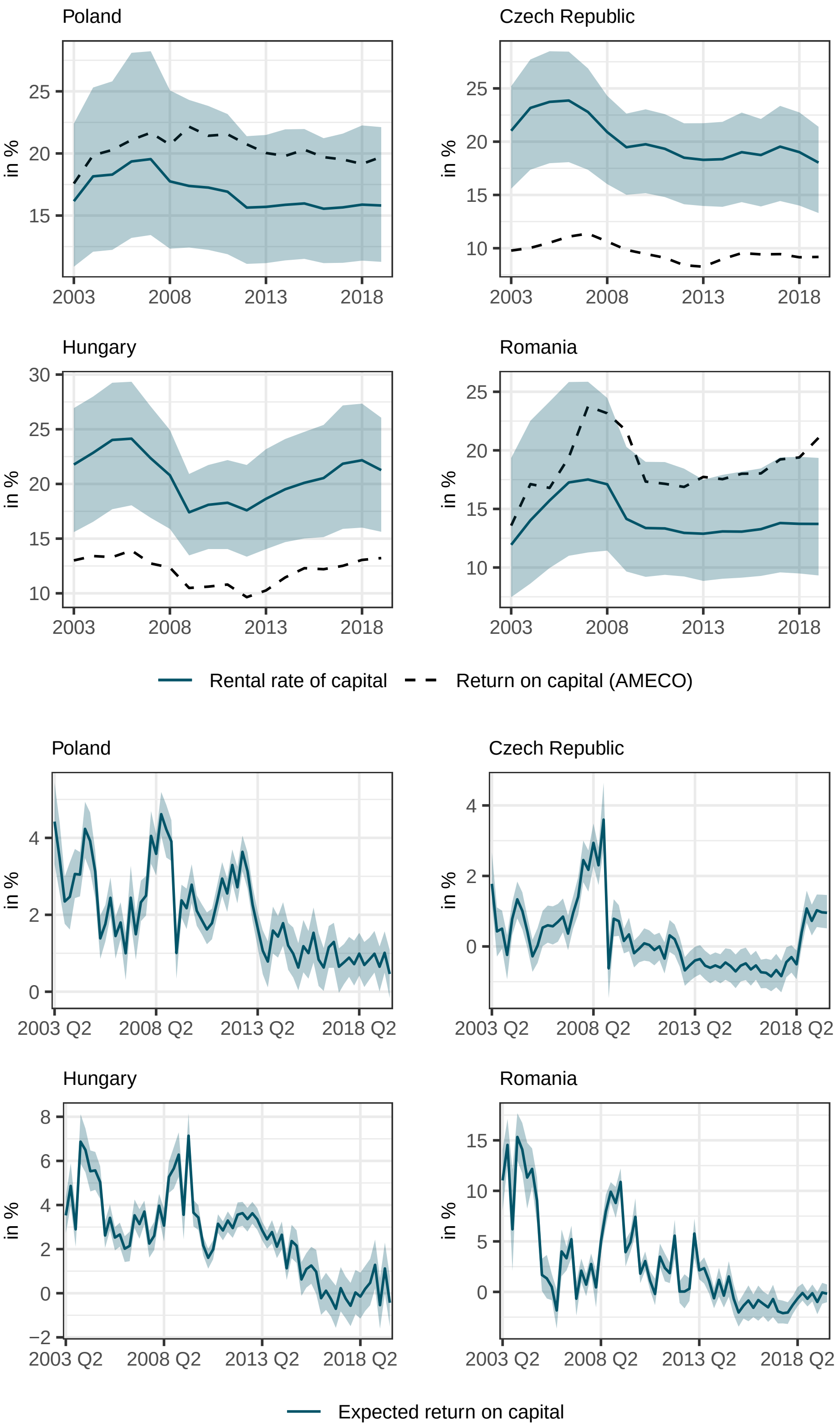

Figure 7: R^* and returns on capital in converging economies. The graph shows median posterior estimates of the rental rate of capital and the (annualized) expected return on capital (including 90 % HPD intervals). The observed annual return on capital is constructed by dividing the series 'Net operating surplus, total economy' by the 'Net capital stock at constant prices; total economy', which is multiplied by the 'Price deflator gross fixed capital formation: total economy' from the AMECO database

	PL	CZ	HU	RO
Natural rate of interest	2.0	0.4	2.7	3.0
Expected return on capital	2.0	0.2	2.5	2.6
Rental rate of capital	17.0	20.3	20.7	14.3
Return on capital (AMECO)	20.3	9.7	12.1	18.7

Table 6: R^* and the return on capital

are close to the natural rates of interest. We believe that this finding is a result of the fact that the real interest rate shares the low-frequency dynamics of the real natural rate, and of the fact that rates of return on capital and real interest rates are necessarily equivalent in our DSGE model at steady state and at a first-order (linear) solution and estimation of the model.[10]

6 Conclusions

Tracking the natural rate of interest derived from structural DSGE models can considerably improve macroeconomic stability (Barsky et al., 2014; Brand et al., 2018). Under certain circumstances (as shown for the U. S. by Justiniano et al. (2013)), setting the real key policy rate equal to r^* closes the welfare-relevant output gap and stabilizes the inflation rate around its target. So far, investigations into the natural rate of interest in emerging economies are underrepresented in the literature. We exploit the historical episode of rapid economic catch-up during almost two decades of formerly centrally-planned economies in Central and Eastern Europe to learn about the natural rate of interest under these specific circumstances. Hence, investigating economic convergence adds a new aspect to the numerous drivers of the decline in r^* that are found in the literature.

Preliminary to estimating the structural model, we present empirical evidence of convergence in a panel of European countries and find that convergence was roughly consistent with the iron law of 2 % convergence per year (Barro, 2012). The empirical evidence helps us to identify initial stages of the capital stocks in the structural model. To build an appropriate environment for catch-up economies, we extend a New Keynesian small open

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[10] Allowing levels of returns on capital and real interest rates to be different would require a modeling framework with, e.g. financial frictions, and/ or estimation methods capable of going beyond linear approximation. We leave this interesting agenda for future research.

economy model in several dimensions and estimate the model in a Bayesian fashion that allows for convergence. The model is estimated for Poland, Czech Republic, Hungary and Romania for the period between 2003 and 2019.

We confirm a secular decline in r^* that ranges between one to six percentage points in the period between 2003 and 2019. Real natural rates of interest appear consistent among the four economies and of reasonable magnitude and average 2.0 %, 0.4 %, 2.7 % and 3.0 % for Poland, Czech Republic, Hungary and Romania. Finally, we show that conflicting signals from the natural rate of interest might arise for monetary policy in emerging economies, when the modelling framework does not properly account for capital accumulation and convergence.

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A Appendix

A.1 Evidence of convergence: Data description and robustness

The sample for the estimation of the growth regressions includes annual data of 42 European countries for the period 1997 to 2017. The list of countries includes Albania (AL), Austria (AT), Belarus (BY), Belgium (BE), Bosnia and Herzegovina (BA), Bulgaria (BG), Croatia (HR), Cyprus (CY), Czech Republic (CZ), Denmark (DK), Estonia (EE), Finland (FI), France (FR), Germany (DE), Greece (GR), Hungary (HU), Iceland (IC), Ireland (IE), Italy (IT), Kosovo (XK), Latvia (LV), Lithuania (LT), Luxembourg (LU), Malta (MT), Macedonia (MK), Moldova (MD), Montenegro (ME), Netherlands (NL), Norway (NO), Poland (PL), Portugal (PT), Romania (RO), Russia (RU), Serbia (RS), Slovakia (SK), Slovenia (SI), Spain (ES), Sweden (SE), Switzerland (SW), Turkey (TR), Ukraine (UA), United Kingdom (UK).

Table 7 lists the description and sources of the variables used in the empirical section 2. To check for robustness, table 8 shows additional conditional convergence regressions.

Variable	**Description and source**
GDP p. c.	gross domestic product (GDP) per capita, at purchasing power parities (PPP), IMF World Economic Outlook (WEO)
GDP p. c. growth	average annual growth rate of GDP p. c., measured as log difference of GDP p. c. between the last year of the 5-year period and the previous year divided by 5, IMF WEO
Investment	gross capital cormation at PPP, per capita, current prices, as share of GDP, 5-year averages, Penn World Table
Population	average 5-year annual growth rate of the total population, IMF WEO
Labor Force	average 5-year annual growth rate of the labor force, IMF International Financial Statistics (IFS)
Institutions	annual average of the world governance indicators for rule of law, regulatory quality, government effectiveness and corruption, World Bank
Schooling	average annual years of schooling (both sexes), Barro and Lee (2013)
Openess	sum of merchandise exports and imports, at PPP, per capita, current prices, as share of GDP, Penn World Table
Credit	average 5-year annual domestic credit to private sector by banks, percent of GDP, World Bank, World Development Indicators
FDI	average 5-year annual foreign direct investment, IMF IFS
Public debt	central government debt, percent of GDP, World Bank, World Development Indicators

Table 7: Evidence of convergence: Data description and sources

Table 8: Absolute and conditional convergence in Europe between 1997 and 2017

	(1)	(2)	(3)	(4)	(5)	(6)
Log GDP p.c.	-0.020***	-0.013	-0.018***	-0.039***	-0.042***	-0.014**
	(0.007)	(0.010)	(0.006)	(0.008)	(0.009)	(0.007)
Investment	0.001**	0.000	0.001**	0.001	0.001**	-0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Population	-0.005	-0.008**	-0.003	-0.001	-0.004	-0.007*
	(0.004)	(0.003)	(0.003)	(0.003)	(0.003)	(0.004)
Labor Force	0.004	0.006***	0.004	0.003	0.004**	0.002
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.004)
Institutions	0.004	0.002	0.003	0.013*	0.008	0.004
	(0.005)	(0.005)	(0.005)	(0.006)	(0.006)	(0.005)
Schooling	0.002*	0.001	0.002*	0.003*	0.005**	0.001
	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.001)
Openness	-0.000	0.000	-0.000	-0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Credit	-0.000**	-0.000*	-0.000**	-0.000**	-0.000**	-0.000**
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
FDI	0.000***	0.000***	0.000***	0.000***	0.000***	0.000***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Public debt	-0.000*	-0.000**	-0.000*	-0.000	-0.000	-0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
2003-2007	0.011**	0.171**	0.012**	0.009*	0.008	0.013***
	(0.005)	(0.079)	(0.005)	(0.005)	(0.005)	(0.005)
2008-2012	-0.026***	0.000	-0.025***	-0.024***	-0.022***	-0.024***
	(0.005)	(0.089)	(0.005)	(0.006)	(0.006)	(0.005)
2013-2017	-0.004	-0.131	-0.003	0.004	0.005	-0.004
	(0.006)	(0.104)	(0.006)	(0.006)	(0.006)	(0.006)
2003-2007 × Log GDP p.c.		-0.016**				
		(0.008)				
2008-2012 × Log GDP p.c.		-0.003				
		(0.009)				
2013-2017 × Log GDP p.c.		0.012				
		(0.010)				
EU × Log GDP p.c.			-0.010*			
			(0.005)			
EU			0.103*			
			(0.054)			
EU_CEE × Log GDP p.c.					-0.022**	
					(0.009)	
EU_CEE					0.205**	
					(0.087)	
Constant	0.204***	0.153	0.183***	0.382***	0.391***	0.176***
	(0.065)	(0.092)	(0.059)	(0.073)	(0.081)	(0.067)
Observations	135	135	135	101	101	135

Cluster-robust standard errors are reported in parentheses. Time fixed effects. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Sources: own calculations, data from IMF WEO and IFS, Penn World Table, World Bank, Barro and Lee (2013).

A.2 Model appendix

A.2.1 Stationarizing the model

Since domestic aggregate total factor productivity has a growing trend A_t, the model is stationarized by defining variables in relation to A_t as follows: $C_{A,t} = \frac{C_t}{A_t^{\frac{\tau+\tau\varphi}{1+\tau\varphi}}}$, $K_{A,t-1} = \frac{K_{t-1}}{A_{t-1}^{\frac{\tau+\tau\varphi}{1+\tau\varphi}}}$, $N_{A,t} = \frac{N_t}{A_t^{\frac{\tau-1}{1+\tau\varphi}}}$, $C_{AH,t} = \frac{C_{H,t}}{A_t^{\frac{\tau+\tau\varphi}{1+\tau\varphi}}}$, $C_{AF,t} = \frac{C_{F,t}}{A_t^{\frac{\tau+\tau\varphi}{1+\tau\varphi}}}$, $C^f_{AH,t} = \frac{C^f_{H,t}}{A_t^{\frac{\tau+\tau\varphi}{1+\tau\varphi}}}$, $I_{A,t} = \frac{I_t}{A_t^{\frac{\tau+\tau\varphi}{1+\tau\varphi}}}$, $Y_{A,t} = \frac{Y_t}{A_t^{\frac{\tau+\tau\varphi}{1+\tau\varphi}}}$, $W_{A,t} = \frac{W_t}{A_t}$, $aux_{A1t} = \frac{aux_{1t}}{A_t^{\frac{\tau+\tau\varphi}{1+\tau\varphi}}}$, $aux_{A2t} = \frac{aux_{2t}}{A_t^{\frac{\tau+\tau\varphi}{1+\tau\varphi}}}$, $mc_t = \frac{MC_t}{P_{H,t}}$. Also, we define real (relative) prices as: $w_{A,t} = \frac{W_{A,t}}{P_t} = \frac{W_t}{A_t P_t}$, $r^K_t = \frac{R^K_t}{P_t}$, $q_t = \frac{Q_t}{P_t}$, $p_{H,t} = \frac{P_{H,\,t}}{P_t}$, $\widetilde{p}_t = \frac{\widetilde{P}_{H,\,t}}{P_{H,t}}$, $\pi_t = \frac{P_t}{P_{t-1}}$, $\pi_{H,t} = \frac{P_{H,t}}{P_{H,t-1}}$, $TOT_t = \frac{P_{F,t}}{P_{H,t}}$, $RER_t = \frac{S_t P^f_t}{P_t}$ and $\widetilde{\Gamma}_A = \Gamma_A^{\frac{\tau+\tau\varphi}{1+\tau\varphi}}$.

Households' intratemporal optimality conditions between home and foreign goods and home demand functions for H- and F-goods become

$$C_{AH,\,t} \;=\; (1-\gamma)\left(p_{H,t}\right)^{-\eta} C_{A,t}, \tag{20}$$

$$C_{AF,\,t} \;=\; \gamma\left(p_{F,t}\right)^{-\eta} C_{A,t}, \tag{21}$$

$$C^f_{AH,\,t} \;=\; \gamma\left(\frac{p_{H,t}}{RER_t}\right)^{-\eta} C^f_{At}. \tag{22}$$

The equations that govern the processes of home and foreign CPI become

$$\frac{1}{p_{H,\,t}} \;=\; \left[(1-\gamma)+\gamma\left(TOT_t\right)^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{23}$$

$$P^f_t = P^f_{F,t} \Longrightarrow \pi^f_t = \pi^f_{F,t} \tag{24}$$

Households' intertemporal conditions become

$$C^{\frac{1}{\tau}}_{A,t} N^{\varphi}_{A,t} \;=\; w_{A,t}, \tag{25}$$

$$C_{A,t}^{-\frac{1}{\tau}} e^{-g_t} = \beta E_t \left[\frac{(1+i_t)}{\pi_{t+1}} C_{A,t+1}^{-\frac{1}{\tau}} e^{-\Delta g_{t+1}} \left(\frac{A_{t+1}}{A_t} \right)^{-\left(\frac{1+\varphi}{1+\tau\varphi}\right)} \right], \tag{26}$$

$$q_t C_{A,t}^{-\frac{1}{\tau}} = \beta E_t C_{A,t+1}^{-\frac{1}{\tau}} \left(\frac{A_{t+1}}{A_t} \right)^{-\left(\frac{1+\varphi}{1+\tau\varphi}\right)} e^{-\Delta g_{t+1}} \left[r_{t+1}^{K} + q_{t+1} (1-\delta) \right], \tag{27}$$

$$\begin{aligned} \left[\begin{array}{c} 1 + \kappa_I q_t \nu_t \left(\frac{I_{A,t}}{I_{A,t-1}} \Gamma_{A,t}^{\frac{\tau+\tau\varphi}{1+\tau\varphi}} - \widetilde{\Gamma}_A \right) \frac{I_{A,t}}{I_{A,t-1}} \Gamma_{A,t}^{\frac{\tau+\tau\varphi}{1+\tau\varphi}} \\ -q_t \nu_t \left[1 - \frac{\kappa_I}{2} \left(\frac{I_t}{I_{t-1}} \Gamma_{A,t}^{\frac{\tau+\tau\varphi}{1+\tau\varphi}} - \widetilde{\Gamma}_A \right)^2 \right] \end{array} \right] &= \kappa_I \beta E_t q_{t+1} \nu_{t+1} \frac{C_{A,t+1}^{-\frac{1}{\tau}}}{C_{A,t}^{-\frac{1}{\tau}}} e^{-\Delta g_{t+1}} \\ & \left(\frac{A_{t+1}}{A_t} \right)^{-\left(\frac{1+\varphi}{1+\tau\varphi}\right)} \left(\frac{I_{A,t+1}}{I_{A,t}} \Gamma_{A,t+1}^{\frac{\tau+\tau\varphi}{1+\tau\varphi}} - \widetilde{\Gamma}_A \right), \end{aligned} \tag{28}$$

$$K_{A,t} = (1-\delta) K_{A,t-1} \left(\frac{A_{t-1}}{A_t} \right)^{\frac{\tau+\tau\varphi}{1+\tau\varphi}} + I_{A,t} \nu_t \left[1 - \frac{\kappa_I}{2} \left(\frac{I_{A,t}}{I_{A,t-1}} \Gamma_{A,t}^{\frac{\tau+\tau\varphi}{1+\tau\varphi}} - \widetilde{\Gamma}_A \right)^2 \right]. \tag{29}$$

The law of one price for H-goods and F-goods and law of motion linking PPI and CPI inflation become:

$$p_{H,t} = RER_t p_{H,t}^{f} \tag{30}$$

$$TOT_t p_{H,t} = RER_t tot_t^{\sigma_q} \tag{31}$$

Note that we add a measurement error shock, tot_t with standard deviation σ_q, to the equation governing the law of one price.

$$\frac{P_{H,t}}{P_{H,t-1}} = \frac{P_{H,t}/P_t}{P_{H,t}/P_{t-1}} \frac{P_t}{P_{t-1}} \tag{32}$$

The market clearing and the risk sharing conditions in stationarized representation are

$$Y_{A,t} = (p_{H,t})^{-\eta} \left[(1-\gamma)(C_{A,t} + I_{A,t}) + \gamma (RER_t)^{\eta} Y_{A,t}^{f} \right], \tag{33}$$

$$C_{A,t} = \upsilon C^{f}_{A,t} \left(RER_t\right)^{\tau}, \tag{34}$$

$$C^{f}_{A,t} = Y^{f}_{A,t}. \tag{35}$$

For the estimation, foreign output $Y^{f}_{A,t}$ is multiplied by the consumption share (0.75) in the above equation. The stationarized equilibrium conditions of domestic firms are written as

$$\frac{1-\alpha}{\alpha} = \frac{w_{A,t} N_{A,t}}{r^{K}_{t}\ K_{A,t-1}} \left(\frac{A_t}{A_{t-1}}\right)^{\frac{\tau+\tau\varphi}{1+\tau\varphi}}, \tag{36}$$

$$mc_t = \frac{\left(r^{K}_{t}\right)^{\alpha} \left(w_{A,t}\right)^{1-\alpha}}{p_{H,t} \alpha^{\alpha} \left(1-\alpha\right)^{1-\alpha}} \frac{1}{A_t}, \tag{37}$$

$$\Delta_t Y_{A,t} = \left(\frac{A_t}{A_{t-1}}\right)^{-\alpha \frac{\tau+\tau\varphi}{1+\tau\varphi}} \left(K_{A,t-1}\right)^{\alpha} \left(N_{A,t}\right)^{1-\alpha}, \tag{38}$$

$$\widetilde{p}_t = \mu_t \frac{aux_{A1,t}}{aux_{A2,t}}, \tag{39}$$

$$aux_{A1t} = mc_t Y_{A,t} + E_t \beta \theta e^{-\frac{g_{t+1}}{g_t}} \left(\frac{C_{A,t+1}}{C_{A,t}}\right)^{-\frac{1}{\tau}} \frac{\left(\pi^{-1}_{H,\,t+1} \pi^{\iota_P}_{H,\,t} \bar{\pi}^{1-\iota_P}_{H}\right)^{1+\epsilon}}{\pi_{t+1}} \left(\frac{A_{t+1}}{A_t}\right)^{\frac{(1+\varphi)(\tau-1)}{1+\tau\varphi}} aux_{A1t+1}, \tag{40}$$

$$aux_{A2t} = Y_{A,t} + E_t \beta \theta e^{-\frac{g_{t+1}}{g_t}} \left(\frac{C_{A,t+1}}{C_{A,t}}\right)^{-\frac{1}{\tau}} \frac{\left(\pi^{-1}_{H,\,t+1} \pi^{\iota_P}_{H,\,t} \bar{\pi}^{1-\iota_P}_{H}\right)^{\epsilon}}{\pi_{t+1}} \left(\frac{A_{t+1}}{A_t}\right)^{\frac{(1+\varphi)(\tau-1)}{1+\tau\varphi}} aux_{A,1t+2}, \tag{41}$$

$$\Delta_t \equiv \left(1-\theta\right) \left(\widetilde{p}_t\right)^{-\epsilon} + \theta \left(\pi^{-1}_{H,\,t} \pi^{\iota_P}_{H,\,t-1} \bar{\pi}^{1-\iota_P}_{H}\right)^{\epsilon} \Delta_{t-1}, \tag{42}$$

$$\widetilde{p}_t = \left[\frac{1 - \theta \left(\pi_{H,t}^{-1} \pi_{H,t-1}^{\iota_P} \bar{\pi}_H^{1-\iota_P} \right)^{\epsilon - 1}}{(1 - \theta)} \right]^{\frac{1}{1-\epsilon}} . \tag{43}$$

Finally, the monetary policy rule becomes

$$\frac{1 + i_t}{1 + i} = \left(\frac{1 + i_{t-1}}{1 + i} \right)^{\rho_i} \left[\left(\frac{\pi_t}{\overline{\pi}} \right)^{\psi_\pi} \left(\frac{Y_{A,t}}{Y_{A,t}^n} \right)^{\psi_Y} \right]^{1-\rho_i} \varepsilon_{i,t}^{\sigma_i}. \tag{44}$$

The system of equations (20) to (44) together with seven exogenous shocks for $\{g_t, a_t, u_t, Y_t^f, \pi_t^f\ \varepsilon_t, tot_t, \nu_t\}$ determines 22 endogenous variables and represents the equilibrium of the small open economy.[11]

A.2.2 Natural output in the small open economy

The flexible price level of output in the domestic economy can be derived from an equivalent system of labor supply, marginal cost, optimal price, production function, resource constraint, risk sharing condition, CPI, and law of one price for foreign goods.[12] Note that all natural variables have superscript n, accordingly the natural rate of interest $r_t^* = R_t^n$.

$$C_{A,t}^{n\frac{1}{\tau}} N_{A,t}^{n\varphi} = w_{A,t}^n, \tag{45}$$

$$\frac{1 - \alpha}{\alpha} = \frac{w_{A,t}^n N_{A,t}^n}{r_t^{Kn}\ K_{A,t-1}^n} \Gamma_{A,t}^{\frac{\tau + \tau\varphi}{1 + \tau\varphi}}, \tag{46}$$

$$mc_t^n = \frac{\left(r_t^{Kn} \right)^\alpha \left(w_{A,t}^n \right)^{1-\alpha}}{p_{H,t}^n \alpha^\alpha (1 - \alpha)^{1-\alpha}} \frac{1}{A_t}, \tag{47}$$

$$\widetilde{p}_t^n = 1 = \frac{\epsilon}{(\epsilon - 1)} (1 - \phi)\, mc_t^n = \mu_t mc_t^n, \tag{48}$$

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[11] Except for equations (21), (24) and (30), which are not needed.

[12] A detailed model appendix is available upon request from the authors.

$$Y^n_{A,t} = \Gamma_{A,t}^{-\alpha\frac{\tau+\tau\varphi}{1+\tau\varphi}} A_t \left(K^n_{A,t-1}\right)^{\alpha} \left(N^n_{A,t}\right)^{1-\alpha} \tag{49}$$

$$Y^n_{A,t} = \left(p^n_{H,t}\right)^{-\eta} \left[\left(1-\gamma\right)\left(C^n_{A,t} + I^n_{A,t}\right) + \gamma \left(RER^n_t\right)^{\eta} Y^{nf}_{A,t}\right] \tag{50}$$

$$C^n_{A,t} = \upsilon C^{nf}_{A,t} \left(RER^n_t\right)^{\tau} \tag{51}$$

$$p^n_{H,\,t} = \left[\left(1-\gamma\right) + \gamma \left(TOT^n_t\right)^{1-\eta}\right]^{\frac{1}{\eta-1}} \tag{52}$$

$$TOT^n_t p^n_{H,t} = RER^n_t tot_n^{\sigma_q} \tag{53}$$

$$C_{A,t}^{n-\frac{1}{\tau}} e^{-g_t} = \beta E_t \left[R^n_t C_{A,t+1}^{n-\frac{1}{\tau}} e^{-g_{t+1}} \left(\frac{A_{t+1}}{A_t}\right)^{-\left(\frac{1+\varphi}{1+\tau\varphi}\right)}\right] \tag{54}$$

$$q^n_t C_{A,t}^{n-\frac{1}{\tau}} = \beta E_t C_{A,t+1}^{n-\frac{1}{\tau}} \Gamma_{A,t+1}^{-\left(\frac{1+\varphi}{1+\tau\varphi}\right)} e^{-\Delta g_{t+1}} \left[r^{Kn}_{t+1} + q^n_{t+1}\left(1-\delta\right)\right] \tag{55}$$

$$\left[\begin{array}{c} 1 + \kappa_I q^n_t \nu_t \left(\frac{I^n_{A,t}}{I^n_{A,t-1}} \Gamma_{A,t}^{\frac{\tau+\tau\varphi}{1+\tau\varphi}} - \widetilde{\Gamma}_A\right) \frac{I^n_{A,t}}{I^n_{A,t-1}} \Gamma_{A,t}^{\frac{\tau+\tau\varphi}{1+\tau\varphi}} \\ -q^n_t \nu_t \left[1 - \frac{\kappa_I}{2} \left(\frac{I^n_{A,t}}{I^n_{A,t-1}} \Gamma_{A,t}^{\frac{\tau+\tau\varphi}{1+\tau\varphi}} - \widetilde{\Gamma}_A\right)^2\right] \end{array}\right] = \left[\begin{array}{c} \kappa_I \beta E_t q^n_{t+1} \nu_{t+1} \frac{C_{A,t+1}^{n-\frac{1}{\tau}}}{C_{A,t}^{n-\frac{1}{\tau}}} e^{-\Delta g_{t+1}} \Gamma_{A,t+1}^{-\left(\frac{1+\varphi}{1+\tau\varphi}\right)} \\ \left(\frac{I^n_{A,t+1}}{I^n_{A,t}} \Gamma_{A,t+1}^{\frac{\tau+\tau\varphi}{1+\tau\varphi}} - \widetilde{\Gamma}_A\right) \end{array}\right] \tag{56}$$

$$K^n_{A,t} - \left(1-\delta\right) K^n_{A,t-1} \left(\Gamma_{A,t}^{-\frac{\tau+\tau\varphi}{1+\tau\varphi}}\right) = I^n_{A,t} \nu_t \left[1 - \frac{\kappa_I}{2} \left(\frac{I^n_{A,t}}{I^n_{A,t-1}} \Gamma_{A,t}^{\frac{\tau+\tau\varphi}{1+\tau\varphi}} - \widetilde{\Gamma}_A\right)^2\right] \tag{57}$$

$$GAP_t = \frac{Y_{A,t}}{Y^n_{A,t}} \tag{58}$$

A.3 Description of observables and additional data

Variable	**Description and source**
Observables:	
Output	Gross domestic product, calendar and seasonally adjusted, chained, 2010 reference year. Eurostat. ESA 2010
Inflation	Harmonized index of consumer prices (HICP) at constant taxes, all-items, seasonally adjusted. ESA 2010
Exchange rate to euro	Local currency per euro. Macrobond Financial AB
Terms of Trade	Price deflator of imports divided by price deflator of exports of goods and services. Eurostat. ESA 2010
Short term interest rates	3-month interbank interest rates. Source: Refinitv Eikon, Czech National Bank
Additional data:	
Import share	Imports of goods and services divided by gross domestic product, current prices, calendar and seasonally adjusted. World Development Indicators, World Bank
Share of imported goods in consumption	World Input-Output Database (WIOD), release 2016 for the period 2000-2014, University of Groningen
Survey inflation expecations	Inflation perceptions, Development of consumer prices in the next 12 months. European Commission (DG ECFIN) Consumer Survey

Table 9: Observables and other: Data description and sources

A.4 Alternative estimation results

Parameter	Prior mean	Post. mean	90 % interval	Prior mean	Post. mean	90 % interval
		Poland			Czech Republic	
τ	0.50	0.73	[0.55,0.93]	0.50	0.71	[0.52,0.91]
γ	0.40	0.38	[0.29,0.48]	0.40	0.35	[0.27,0.42]
η	1.00	0.76	[0.5,1]	1.00	1.43	[1.1,1.75]
φ	1.50	0.76	[0.29,1.23]	1.50	1.16	[0.56,1.74]
θ	0.66	0.09	[0.02,0.15]	0.66	0.13	[0.04,0.23]
κ_I	4.00	3.62	[2.54,4.68]	4.00	4.41	[3.08,5.77]
δ	0.03	0.03	[0.01,0.04]	0.03	0.04	[0.03,0.05]
α	0.30	0.23	[0.2,0.25]	0.30	0.25	[0.21,0.28]
ψ_Y	0.50	0.12	[0.04,0.21]	0.50	0.08	[0.02,0.14]
ψ_π	1.50	3.04	[2.18,3.87]	1.50	2.62	[1.89,3.35]
ρ_i	0.50	0.82	[0.76,0.89]	0.50	0.83	[0.76,0.89]
r_A	1.00	0.80	[0.54,1.04]	1.00	0.68	[0.45,0.89]
π_A	4.00	2.14	[1.1,3.21]	4.00	1.13	[0.51,1.76]
Γ_Q	0.75	0.44	[0.26,0.62]	0.75	0.10	[0,0.19]
$consTOT$	0.00	-0.07	[-0.26,0.09]	0.00	-0.05	[-0.13,0.03]
$consNER$	0.00	0.00	[-0.16,0.17]	0.00	-0.01	[-0.17,0.15]
ι_P	0.50	0.48	[0.31,0.65]	0.50	0.47	[0.31,0.63]
		Hungary			Romania	
τ	0.50	0.86	[0.75,0.98]	0.50	0.62	[0.42,0.82]
γ	0.30	0.27	[0.2,0.34]	0.30	0.14	[0.08,0.2]
η	1.00	1.16	[0.83,1.48]	1.00	1.07	[0.74,1.41]
φ	1.50	1.10	[0.59,1.61]	1.50	1.31	[0.66,1.94]
θ	0.66	0.36	[0.25,0.46]	0.66	0.61	[0.44,0.8]
κ_I	4.00	3.12	[1.66,4.65]	4.00	3.11	[2.15,4.18]
δ	0.03	0.03	[0.01,0.04]	0.03	0.02	[0.01,0.02]
α	0.30	0.23	[0.2,0.26]	0.30	0.23	[0.2,0.25]
ψ_Y	0.50	0.31	[0.14,0.47]	0.50	0.25	[0.1,0.39]
ψ_π	1.50	2.12	[1.48,2.74]	1.50	2.05	[1.43,2.64]
ρ_i	0.50	0.77	[0.69,0.85]	0.50	0.71	[0.62,0.81]
r_A	1.00	0.90	[0.62,1.18]	1.00	0.92	[0.63,1.22]
π_A	4.00	2.43	[0.82,3.99]	4.00	2.27	[0.79,3.63]
Γ_Q	0.75	0.41	[0.26,0.56]	0.75	0.54	[0.32,0.74]
$consTOT$	0.00	-0.05	[-0.15,0.04]	0.00	0.15	[0,0.3]
$consNER$	0.00	0.00	[-0.16,0.16]	0.00	0.01	[-0.15,0.17]
ι_P	0.50	0.44	[0.28,0.61]	0.50	0.39	[0.22,0.55]

Table 10: Posterior estimation results of the model with alternative inital k: Posterior mean, lower and upper bound of 90 % HPD interval.

Parameter	Prior mean	Post. mean	90 % interval	Prior mean	Post. mean	90 % interval
		Poland			Czech Republic	
ρ_z	0.20	0.06	[0.01,0.11]	0.20	0.04	[0.01,0.07]
ρ_{y^f}	0.90	0.97	[0.95,1]	0.90	0.96	[0.94,0.98]
ρ_{π^f}	0.80	0.50	[0.35,0.65]	0.80	0.54	[0.38,0.68]
ρ_g	0.20	0.25	[0.06,0.43]	0.20	0.28	[0.07,0.48]
ρ_u	0.80	0.87	[0.78,0.96]	0.80	0.82	[0.73,0.92]
ρ_ν	0.80	0.63	[0.47,0.79]	0.80	0.75	[0.64,0.86]
σ_i	0.50	0.26	[0.19,0.33]	0.50	0.21	[0.17,0.26]
σ_z	1.00	1.54	[1.07,1.99]	1.00	1.51	[1.19,1.82]
σ_{y^f}	1.50	2.19	[1.54,2.82]	1.50	1.17	[0.92,1.43]
σ_{π^f}	0.55	3.91	[3.31,4.47]	0.55	2.32	[1.96,2.66]
σ_g	0.25	0.17	[0.11,0.23]	0.25	0.16	[0.11,0.2]
σ_u	0.55	2.03	[1.14,2.91]	0.55	1.17	[0.71,1.6]
σ_q	1.00	0.45	[0.31,0.59]	1.00	0.40	[0.3,0.49]
σ_ν	0.25	8.32	[6.66,10]	0.25	2.98	[2.04,3.89]
		Hungary			Romania	
ρ_z	0.20	0.06	[0.02,0.11]	0.20	0.16	[0.02,0.31]
ρ_{y^f}	0.90	0.91	[0.86,0.97]	0.90	0.95	[0.92,0.98]
ρ_{π^f}	0.80	0.47	[0.32,0.62]	0.80	0.63	[0.46,0.81]
ρ_g	0.20	0.23	[0.05,0.41]	0.20	0.21	[0.04,0.36]
ρ_u	0.80	0.96	[0.93,0.99]	0.80	0.78	[0.62,0.96]
ρ_ν	0.80	0.40	[0.27,0.53]	0.80	0.57	[0.44,0.69]
σ_i	0.50	0.45	[0.33,0.56]	0.50	0.57	[0.45,0.68]
σ_z	1.00	1.37	[1.13,1.61]	1.00	2.41	[1.63,3.26]
σ_{y^f}	1.50	1.06	[0.88,1.25]	1.50	1.49	[1.11,1.88]
σ_{π^f}	0.55	3.04	[2.58,3.48]	0.55	1.61	[0.99,2.18]
σ_g	0.25	0.19	[0.12,0.26]	0.25	0.25	[0.12,0.38]
σ_u	0.55	1.18	[0.76,1.61]	0.55	3.79	[2.4,5.15]
σ_q	1.00	0.45	[0.33,0.56]	1.00	1.77	[1.33,2.19]
σ_ν	0.25	12.39	[7.05,18.41]	0.25	16.59	[13.09,20]

Table 11: Posterior estimation results of the model with alternative inital k: Shock parameters. Posterior mean, lower and upper bound of 90 % HPD interval.

Parameter	Prior mean	Post. mean	90 % interval	Prior mean	Post. mean	90 % interval
	Poland			Czech Republic		
τ	0.50	0.89	[0.8,0.98]	0.50	0.91	[0.83,0.99]
γ	0.30	0.08	[0.07,0.09]	0.30	0.18	[0.12,0.23]
η	1.00	0.80	[0.53,1.05]	1.00	1.02	[0.7,1.32]
φ	1.50	1.20	[0.51,1.83]	1.50	1.41	[0.73,2.13]
θ	0.66	0.36	[0.24,0.49]	0.66	0.19	[0.09,0.29]
ψ_Y	0.50	0.75	[0.22,1.24]	0.50	0.74	[0.15,1.28]
ψ_π	1.50	1.47	[1.03,1.88]	1.50	1.16	[1,1.36]
ρ_i	0.50	0.78	[0.7,0.85]	0.50	0.71	[0.64,0.79]
r_A	1.00	0.75	[0.52,0.99]	1.00	0.66	[0.46,0.87]
π_A	4.00	1.87	[1,2.73]	4.00	1.42	[0.64,2.12]
Γ_Q	0.75	0.46	[0.34,0.6]	0.75	0.07	[0,0.14]
$consTOT$	0.00	0.12	[0.01,0.23]	0.00	-0.02	[-0.1,0.05]
$consNER$	0.00	0.00	[-0.17,0.16]	0.00	-0.01	[-0.18,0.15]
ι_P	0.50	0.46	[0.29,0.63]	0.50	0.46	[0.3,0.62]
	Hungary			Romania		
τ	0.50	0.90	[0.83,0.99]	0.50	0.87	[0.76,0.97]
γ	0.30	0.25	[0.18,0.32]	0.30	0.18	[0.11,0.25]
η	1.00	1.06	[0.73,1.38]	1.00	0.84	[0.57,1.11]
φ	1.50	1.68	[0.85,2.5]	1.50	1.69	[0.83,2.52]
θ	0.66	0.27	[0.17,0.37]	0.66	0.14	[0.03,0.25]
ψ_Y	0.50	0.52	[0.13,0.91]	0.50	0.55	[0.12,0.95]
ψ_π	1.50	2.54	[1.85,3.21]	1.50	2.34	[1.86,2.81]
ρ_i	0.50	0.75	[0.66,0.84]	0.50	0.55	[0.42,0.69]
r_A	1.00	0.89	[0.6,1.17]	1.00	0.91	[0.62,1.2]
π_A	4.00	3.09	[2.38,3.82]	4.00	4.55	[3.66,5.46]
Γ_Q	0.75	0.47	[0.32,0.62]	0.75	0.69	[0.5,0.88]
$consTOT$	0.00	-0.06	[-0.14,0.01]	0.00	0.19	[0.04,0.34]
$consNER$	0.00	0.01	[-0.15,0.18]	0.00	0.03	[-0.13,0.19]
ι_P	0.50	0.46	[0.3,0.63]	0.50	0.48	[0.31,0.65]

Table 12: Posterior estimation results of the model without capital: Posterior mean, lower and upper bound of 90 % HPD interval.

Parameter	Prior mean	Post. mean	90 % interval	Prior mean	Post. mean	90 % interval
	Poland			Czech Republic		
ρ_z	0.20	0.11	[0.03,0.18]	0.20	0.06	[0.01,0.11]
ρ_{y^f}	0.90	0.91	[0.84,0.98]	0.90	0.92	[0.86,0.98]
ρ_{π^f}	0.80	0.56	[0.41,0.71]	0.80	0.58	[0.43,0.73]
ρ_g	0.20	0.23	[0.06,0.41]	0.20	0.31	[0.08,0.52]
ρ_u	0.80	0.90	[0.85,0.95]	0.80	0.93	[0.88,0.97]
σ_i	0.50	0.22	[0.18,0.27]	0.50	0.20	[0.16,0.24]
σ_z	1.00	1.02	[0.83,1.22]	1.00	1.14	[0.96,1.32]
σ_{y^f}	1.50	1.26	[0.98,1.55]	1.50	0.86	[0.72,1]
σ_{π^f}	0.55	3.18	[2.64,3.73]	0.55	2.16	[1.79,2.53]
σ_g	0.25	0.19	[0.12,0.26]	0.25	0.15	[0.1,0.19]
σ_u	0.55	1.42	[0.85,1.96]	0.55	0.72	[0.41,1.01]
σ_q	1.00	1.84	[1.48,2.22]	1.00	0.61	[0.45,0.77]
	Hungary			Romania		
ρ_z	0.20	0.11	[0.04,0.17]	0.20	0.08	[0.02,0.14]
ρ_{y^f}	0.90	0.91	[0.85,0.98]	0.90	0.97	[0.95,0.99]
ρ_{π^f}	0.80	0.47	[0.32,0.63]	0.80	0.62	[0.45,0.78]
ρ_g	0.20	0.86	[0.82,0.9]	0.20	0.77	[0.7,0.85]
ρ_u	0.80	0.81	[0.67,0.96]	0.80	0.79	[0.63,0.95]
σ_i	0.50	0.44	[0.31,0.57]	0.50	0.70	[0.52,0.88]
σ_z	1.00	1.03	[0.87,1.19]	1.00	1.48	[1.23,1.71]
σ_{y^f}	1.50	0.93	[0.77,1.08]	1.50	1.57	[1.24,1.89]
σ_{π^f}	0.55	3.07	[2.62,3.54]	0.55	1.94	[1.42,2.43]
σ_g	0.25	1.46	[1.08,1.83]	0.25	1.98	[1.49,2.46]
σ_u	0.55	0.52	[0.29,0.75]	0.55	0.58	[0.29,0.89]
σ_q	1.00	0.44	[0.32,0.54]	1.00	1.55	[1.19,1.9]

Table 13: Posterior estimation results of the model without capital: Standard deviations of shocks. Posterior mean, lower and upper bound of 90 % HPD interval.