Emotions in tournaments

Kräkel, Matthias

Accepted Manuscript

Title: Emotions in Tournaments

Author: Matthias Kräkel

Emotions in Tournaments*

Matthias Kräkel, *University of Bonn***

We introduce a concept of emotions that emerge when agents compare their own performance with the performances of other agents. Assuming heterogeneity among the agents the interplay of emotions and incentives is analyzed within the framework of rank-order tournaments, which are frequently used in practice. Tournaments seem to be an appropriate starting point for this concept because a tournament induces incentives by making agents compare themselves with their opponents. We identify certain conditions under which the principal benefits from emotional agents. Furthermore, the concept of emotions is used to explain the puzzling findings on the oversupply of effort in experimental tournaments.

Key words: contest design, emotions, tournaments, unfair contests.

JEL classification: J3, M5.

** Matthias Kräkel, Department of Economics, BWL II, University of Bonn, Adenauerallee 24-42, D-53113 Bonn, Germany, e-mail: m.kraekel@uni-bonn.de, phone: +49-228-739211, fax: +49-228-739210.

1

1 Introduction

Emotions are a natural ingredient of human beings. In particular, when evaluating possible consequences of their decisions, people take emotions such as disappointment, frustration, joy or pride into account. Hence, an economic decision maker should also incorporate possible emotions into his objective function. Moreover, the experimental findings of Bosman and van Winden (2002) on emotional hazard point out that emotions play an important role in real decision making. However, as Elster (1998) and Loewenstein (2000) complain, economists (with some exceptions[1]) do not pay attention to emotions when modelling economic behavior although introducing emotions may "help us explain behavior for which good explanations seem to be lacking" (Elster, 489).

In this paper, emotions are introduced into the theory of rank-order tournaments. In a (rank-order) tournament, at least two agents compete against each other for given prizes. The agent with the best performance receives the winner prize, the second best agent gets the second highest prize, and so on. There exist many examples for tournaments in economics.[2] They can be observed between salesmen (e.g., Mantrala et al. 2000), in broiler production (Knoeber and Thurman 1994), and in hierarchical firms when people compete for job promotion (e.g., Baker et al. 1994, Eriksson 1999, Bognanno 2001). Basically, corporate tournaments will always be created if relative performance evaluation is linked to monetary consequences for the

[1]See, for example, Hirshleifer (1987) on emotions as guarantors of threats and promises, Kandel and Lazear (1992) on shame and guilt in the context of peer pressure, and Mui (1995) on envy.

[2]For a theoretical analysis of tournaments see Lazear and Rosen (1981), Green and Stokey (1983), Nalebuff and Stiglitz (1983), and Rosen (1986).

2

employees. Hence, forced-ranking or forced-distribution systems, in which supervisors have to rate their subordinates according to a given number of different grades, also belong to the class of tournament incentive schemes (see, for example, Murphy 1992 on forced ranking at Merck). Boyle (2001) reports that about 25 per cent of the so-called Fortune 500 companies utilize forced-ranking systems to tie pay to performance (e.g., Cisco Systems, Intel, General Electric).

In the following, we will consider emotions that will emerge if agents compare their own performance with the performances of other agents who participate in the same tournament. Typically, agents feel joy or pride (*positive emotions*) when outperforming their opponents, whereas they are disappointed (*negative emotions*) when falling behind them. Here, emotions will be called positive (negative) if they lead to an increase (a decrease) of an agent's utility. Such positive and negative emotions should be strongest in so-called "unfair tournaments" (O'Keeffe et al. 1984) in which a more able agent (favorite) competes against a less able one (underdog); in such a tournament, the latter's probability of winning is less than $1/2$ prior to the contest. If the underdog unexpectedly wins against the favorite he will feel joy or pride so that his subjectively *perceived* winner prize (in monetary terms) should be higher than the monetary winner prize offered by the principal. However, if a clear favorite loses against an underdog, the favorite may be very disappointed since his likelihood of winning strictly exceeded $1/2$. Thus, his perceived loser prize (in monetary) terms should be even lower than the official loser prize in this case.

The aim of the paper is twofold. First, it will be emphasized that emotions are not always detrimental, as pointed out by the experiments on emotional hazard and the model by Mui on envy, for example. We can show under

3

which conditions emotions are beneficial for a principal who maximizes expected profits. In particular, we can show that an agent's equilibrium effort increases in both his positive and his negative emotions. If the agents' cost-of-effort function is sufficiently steep, overall incentives of both agents will rise due to positive and negative emotions. However, the impact of increased heterogeneity on incentives is ambivalent. On the one hand, a favorite (an underdog) has a small (large) perceived loser (winner) prize, and the large spread between perceived winner and loser prize of both agents even rises in the ability difference. Hence, a larger ability difference of the agents will enhance emotions and, therefore, also incentives in equilibrium (emotion effect). On the other hand, a larger ability difference may lead to more or less uneven competition between the agents depending on their different degree of risk aversion and their emotions (competition effect). Incentives increase if the competition becomes more even but decrease if it becomes more uneven. Altogether, if the emotion and competition effects work into the same direction or the emotion effect dominates the competition effect, equilibrium incentives will rise in the ability difference or unfairness of the tournament. This finding is surprising since standard tournament results show that the principal should avoid unfair tournaments between heterogeneous agents since their equilibrium efforts are decreasing in the ability difference. Finally, we will show that the principal may strictly benefit from both positive and negative emotions. This will be the case if tournament prizes are exogenously given or if the principal endogenously chooses tournament prizes but does not have to pay for the emotional incentives since an agent earns a positive rent. Then these extra incentives will only reduce the agent's rent.

Second, the paper seizes the suggestion made by Elster and utilizes emotions to explain empirical findings that contradict standard economic theory.

4

There exist diverse experimental findings on asymmetric tournaments that are puzzling as they show that players significantly oversupply effort compared to equilibrium effort levels (Bull et al. 1987, Weigelt et al. 1989, Schotter and Weigelt 1992). By using the concept of emotions these results can easily be explained.

There are parallels to other papers on incentives that depart from the standard assumption that agents solely care for their absolute incomes. Similar to the notion of pride, Fershtman et al. (2003a, 2003b) consider a concept of so-called competitive preferences in which a player derives utility from being ahead. They apply their concept to standard individualistic incentive schemes. If we applied this concept to tournaments, the subjective winner prize of each contestant would be larger than the monetary winner prize, irrespective of whether agents are homogeneous or heterogeneous. Hence, under that concept standard tournament results will qualitatively remain the same. One would only have to redefine the given tournament prizes as subjective prizes. However, in this paper, we assume that emotions that emerge when comparing one's own performance with the performances of co-workers will depend on the ability difference and, therefore, on the type of co-worker.

There are also parallels to the status motive in competition (e.g., Frank and Cook 1996, 112-114). Moldovanu et al. (2007) explicitly model contests for status when agents care about their relative position in a hierarchy. They discuss how a principal can utilize certain status categories in order to influence the agents' effort choices in the contest. In particular, they show that the highest status category will contain a unique element if agents are solely interested in status. If status categories are endogenously determined by monetary tournament prizes, the optimal partition of the hierarchy contains only two categories.

5

Finally, the emotion approach can be compared to prospect theory developed by Kahneman and Tversky (1979). According to prospect theory, individuals evaluate the consequences of their decisions in relation to a certain reference point. Moreover, their value functions are S-shaped, being concave for gains and convex for losses. When applying this theory to our tournament problem with heterogeneous agents we can imagine that a favorite who is more likely to win has a higher reference point than the underdog who is expected to lose.

The paper is organized as follows. In the next section, the model is introduced. In Section 3, first the tournament game at the second stage is solved for given tournament prizes. Then we consider the first stage where the principal chooses the optimal prizes. In Section 4, the informational assumptions and an alternative view on emotions will be discussed. Section 5 concludes.

2 The Model

We consider a tournament with two risk averse contestants and a risk neutral principal.[3] Each agent's observable (but unverifiable) performance or output can be described by the production function $q_i = e_i + a_i + \varepsilon_i$ $(i = 1, 2)$.[4] e_i denotes endogenous effort chosen by agent i and a_i agent i's exogenous

---

[3]Most of the assumptions follow the standard tournament model by Lazear and Rosen.

[4]By assuming an observable but unverifiable performance signal we can exclude standard individualistic incentive schemes such as piece rates that would not work in this context whereas tournament incentives will still hold; see Malcomson (1984). Note that this commitment argument also forces the principal to fix in advance general tournament prizes that do not depend on the identity of the winner.

6

ability, which is assumed to be common knowledge.[5] Let Δa stand for the ability difference between j and i, $a_j - a_i$. Without loss of generality, we assume $\Delta a \geq 0$ so that Δa can be used as a measure for heterogeneity in the tournament. The variable ε_i denotes individual noise, also assumed to be exogenous. The noise variables ε_1 and ε_2 are identically and independently distributed with density $g(\varepsilon)$ and cumulative distribution function $G(\varepsilon)$. Let $f(\cdot)$ denote the density and $F(\cdot)$ the cumulative distribution function of the composed random variable $\varepsilon_j - \varepsilon_i$ $(i \neq j)$. It is assumed that $f(\cdot)$ is unimodal with mode zero.[6] The principal can observe only realized output q_i but none of its components. Exerting effort entails costs on an agent, which are described by the function $c(e_i)$ with $c(0) = 0$, $c'(e_i) > 0$ and $c''(e_i) > 0$. The reservation value of each agent is $\bar{u} \geq 0$.

In the tournament, the two agents i and j compete for the monetary prizes w_H and w_L with $w_H > w_L$. If $q_i > q_j$, agent i will receive the high winner prize w_H, whereas agent j will get the loser prize w_L $(i \neq j)$. This paper departs from the standard tournament literature by assuming that agents have *perceived prizes* that may differ from the tournament prizes w_H and w_L. In particular, we assume that agent i $(i = 1, 2)$ may feel positive emotions, η_i^+, when winning the tournament and negative emotions, η_i^-, in case of losing. Positive emotions increase an agent's utility whereas negative emotions lead to decreased utility. Both emotions are assumed to be common knowledge.

On the one hand, if, for example, a clear underdog i wins against a

---

[5]For modeling heterogeneity in ability, we adopt the additive model of Meyer and Vickers (1997), Holmström (1999), and Höffler and Sliwka (2003).

[6]For example, if ε_1 and ε_2 are uniformly distributed over $[-\bar{\varepsilon}, \bar{\varepsilon}]$ (normally distributed), the convolution $f(\cdot)$ will be a triangular distribution over $[-2\bar{\varepsilon}, 2\bar{\varepsilon}]$ (normal distribution) with mean zero.

favorite j the underdog may feel joy or pride since, given identical efforts, his likelihood of winning is less than $1/2$ prior to the contest. In this situation, his subjectively perceived winner prize (in monetary terms) should be higher than the winner prize w_H. On the other hand, if a clear favorite j loses against an underdog i, the favorite may be disappointed since his probability of winning strictly exceeded $1/2$. Hence, his perceived loser prize (in monetary) terms should be even lower than w_L in this case.

Emotions are modelled by assuming that each agent i has a subjectively perceived winner prize, $w_i^+ = w_i^+ \left(w_H, \eta_i^+ \left(\Delta a \right) \right)$, and a perceived loser prize, $w_i^- = w_i^- \left(w_L, \eta_i^- \left(\Delta a \right) \right)$, with $\partial w_i^+ / \partial w_H > 0$ and $\partial w_i^- / \partial w_L > 0$ $(i = 1, 2)$. Moreover, let $\partial w_i^+ / \partial \eta_i^+ > 0$ indicate positive emotions from winning and $\partial w_i^- / \partial \eta_i^- < 0$ negative emotions from losing. Finally, we assume $w_i^+ > w_i^-$ $(i = 1, 2)$ so that agents have incentives to win the tournament. The notations $\eta_i^+ \left(\Delta a \right)$ and $\eta_i^- \left(\Delta a \right)$ allow both types of emotions to depend on the ability difference of the agents or, in other words, on the degree of heterogeneity.

As mentioned above, agents are assumed to be risk averse. In particular, each agent i has a preference function $U_i \left(L_i, e_i \right) = E \left[u_i \left(\tilde{w}_i \right) - c \left(e_i \right) \right]$ $\left(\tilde{w}_i \in \left\{ w_i^-, w_i^+ \right\} \right)$, E denotes the expectation operator with respect to $\tilde{w}_i$) that is additively separable into the utility from facing the monetary lottery $L_i = \left(w_i^+, p_i; w_i^-, 1 - p_i \right)$ with p_i denoting i's probability of winning the tournament and the disutility of effort, $c \left(e_i \right)$.[7] The utility function $u_i \left(\tilde{w}_i \right)$ is assumed to be strictly concave with $u_i \left(0 \right) = 0$, $u_i' \left(\tilde{w}_i \right) > 0$ and $u_i'' \left(\tilde{w}_i \right) < 0$.

We consider a two-stage game. At the first stage, the principal chooses

---

[7]To simplify matters, the assumption of an additively separable utility function is often used in principal-agent models; see, for example, Mas-Colell et al. (1995, 480). For an overview of expected and non-expected utility models of preferences see Machina (1987).

tournament prizes (w_L, w_H) in order to maximize expected net profits (i.e. expected outputs minus prizes). At the second stage, each agent maximizes his preference function $U_i(L_i, e_i)$ for given tournament prizes.

3 Results

We solve the game by working backwards, beginning with the tournament competition between the two agents after prizes have been fixed. Then we go to stage 1 where the principal anticipates the agents' equilibrium behavior in the tournament and chooses his optimal tournament prizes.

3.1 Incentives at the Tournament Stage

In this subsection, we analyze the tournament game at stage 2 for given prizes w_H and w_L. Agent i's objective function can be written as

$$U_i(L_i, e_i) = u_i(w_i^+) p_i + u_i(w_i^-)(1 - p_i) - c(e_i)$$
$$= u_i(w_i^-) + \Delta u_i(w_i^+, w_i^-) p_i - c(e_i) \tag{1}$$

with $\Delta u_i(w_i^+, w_i^-) := u_i(w_i^+) - u_i(w_i^-)$ and $p_i = \text{prob}\{q_i > q_j\} = F(e_i - e_j - \Delta a)$. In analogy, we obtain for agent j

$$U_j(L_j, e_j) = u_j(w_j^-) + \Delta u_j(w_j^+, w_j^-)(1 - p_i) - c(e_j) \tag{2}$$

with $\Delta u_j(w_j^+, w_j^-) := u_j(w_j^+) - u_j(w_j^-)$. Hence, each agent realizes his utility from a perceived loser prize as a fall-back position and earns the additional utility spread $\Delta u_i(w_i^+, w_i^-)$ or $\Delta u_j(w_j^+, w_j^-)$ in case of winning the tournament. In any case, each agent has to bear his costs from exerting effort.

9

Given their perceived tournament prizes, the two agents choose their efforts in order to maximize (1) and (2), respectively. If an equilibrium in pure strategies exists, it will be described by the first-order conditions[8]

$$\Delta u_i \left(w_i^+, w_i^- \right) f \left(e_i^* - e_j^* - \Delta a \right) - c' \left(e_i^* \right) = 0 \tag{3}$$

$$\Delta u_j \left(w_j^+, w_j^- \right) f \left(e_i^* - e_j^* - \Delta a \right) - c' \left(e_j^* \right) = 0. \tag{4}$$

Hence, in equilibrium each agent chooses an effort level that equates marginal expected utility from winning the tournament and marginal costs from exerting effort. The flatter the density $f(\cdot)$ (i.e. the higher exogenous production risk) and the steeper the cost function, the lower will be the equilibrium effort of an agent. Moreover, individual incentives rise in the winner prize w_H (as $\partial w_i^+ / \partial w_H > 0$ and $\partial w_j^+ / \partial w_H > 0$) and decrease in the loser prize w_L (as $\partial w_i^- / \partial w_L > 0$ and $\partial w_j^- / \partial w_L > 0$). Intuitively, each agent receives w_L for sure, either when losing or as part of the winner prize in case of winning, reducing incentives. Contrary to w_L, realizing extra utility from winning and receiving w_H fosters incentives.

The impact of emotions on the agents' effort choices can be investigated by applying the general implicit-function rule to the system of equations (3) and (4) with $\eta_i = \eta_i^+, \eta_i^-$ and $\eta_j = \eta_j^+, \eta_j^-$:[9]

$$\frac{\partial e_i^*}{\partial \eta_i} = -\frac{\partial \Delta u_i}{\partial \eta_i} \frac{\bar{f}}{|J|} \underbrace{\left[-\Delta u_j \bar{f}' - c''(e_j^*) \right]}_{< 0 \text{ due to SOC}_j} \tag{5}$$

$$\frac{\partial e_j^*}{\partial \eta_i} = \frac{\partial \Delta u_i}{\partial \eta_i} \frac{\bar{f}}{|J|} \Delta u_j \bar{f}' \tag{6}$$

[8]To guarantee existence, $f(\cdot)$ has to be sufficiently flat, the utility spread sufficiently small and/or $c(\cdot)$ sufficiently steep; see Lazear and Rosen (1981, 845) and Nalebuff and Stiglitz (1983), for example.

[9]"SOC$_i$" and "SOC$_j$" denote the second-order conditions of agents i and j, respectively.

10

$$\frac{\partial e_i^*}{\partial \eta_j} = -\frac{\partial \Delta u_j}{\partial \eta_j} \frac{\bar{f}}{|J|} \Delta u_i \bar{f}' \tag{7}$$

$$\frac{\partial e_j^*}{\partial \eta_j} = -\frac{\partial \Delta u_j}{\partial \eta_j} \frac{\bar{f}}{|J|} \underbrace{[\Delta u_i \bar{f}' - c''(e_i^*)]}_{< \, 0 \text{ due to } SOC_i} \tag{8}$$

where $\Delta u_i := \Delta u_i \left(w_i^+, w_i^- \right)$, $\Delta u_j := \Delta u_j \left(w_j^+, w_j^- \right)$, $\bar{f} := f \left(e_i^* - e_j^* - \Delta a \right)$, and

$$|J| = \underbrace{(\Delta u_i \bar{f}' - c''(e_i^*))}_{< \, 0 \text{ due to } SOC_i} \underbrace{(-\Delta u_j \bar{f}' - c''(e_j^*))}_{< \, 0 \text{ due to } SOC_j} + \Delta u_i \Delta u_j \left[\bar{f}' \right]^2 > 0$$

denotes the Jacobian determinant. Since the density function $f(\cdot)$ has a unique mode at zero so that it is monotonically increasing (decreasing) at the left-hand side (right-hand side), we obtain the following results:

Proposition 1 *Let tournament prizes be exogenously given, $\eta_i = \eta_i^+, \eta_i^-$ and $\eta_j = \eta_j^+, \eta_j^-$. Then $\frac{\partial e_i^*}{\partial \eta_i} > 0$ and $\frac{\partial e_j^*}{\partial \eta_j} > 0$. Moreover, if $e_i^* < e_j^* + \Delta a$, then $\frac{\partial e_j^*}{\partial \eta_i} > 0$ and $\frac{\partial e_i^*}{\partial \eta_j} < 0$; if $e_i^* > e_j^* + \Delta a$ the opposite will hold.*

Proposition 1 shows that each agent's equilibrium effort is always increasing in his own emotions. However, the spillover effects on the other agent's incentives depend on whether the equilibrium $\left(e_i^*, e_j^* \right)$ is located at the left-hand side (LHS) or at the right-hand side (RHS) of $f(\cdot)$. Consider, for example, an increase of η_i that directly motivates i to exert more effort. When we are at the LHS, this increase of e_i^* makes the uneven competition less uneven. Therefore, overall incentives of both agents rise leading to $\frac{\partial e_j^*}{\partial \eta_i} > 0$. However, if we are at the RHS and i's effort increases due to an increase of η_i the uneven competition will become more uneven so that overall incentives are reduced and we have $\frac{\partial e_j^*}{\partial \eta_i} < 0$. From (5)–(8) we can see that the overall incentive effect of i's or j's emotions, respectively, is positive

11

if and only if

$$\frac{\partial e_i^*}{\partial \eta_i} + \frac{\partial e_j^*}{\partial \eta_i} > 0 \Leftrightarrow c''(e_j^*) > -2\Delta u_j \bar{f}' \tag{9}$$

$$\frac{\partial e_i^*}{\partial \eta_j} + \frac{\partial e_j^*}{\partial \eta_j} > 0 \Leftrightarrow c''(e_i^*) > 2\Delta u_i \bar{f}'. \tag{10}$$

The first inequality is always satisfied at the LHS, whereas the second inequality holds for all equilibria at the RHS. Altogether, we have the following result:

Corollary 1 *If the cost function is sufficiently convex, the overall incentive effect of agents' emotions will be positive.*

Note that a sufficiently convex cost function also supports the existence of pure-strategy equilibria at the tournament stage. However, the second-order conditions do not imply inequalities (9) and (10). The intuition for the finding of the corollary is the following: if we have a very steep cost function, effort incentives are rather low. In this situation, the spillover effects mentioned above and their impact on the competitiveness of the tournament are negligible so that the primary incentive effects from one's own incentives are always dominant.[10]

It is interesting to apply the results of Proposition 1 and Corollary 1 to the experimental findings of Weigelt et al. (1989) and Schotter and Weigelt (1992).[11] The authors conduct several experiments on "unfair" tournaments in the notion of O'Keeffe et al.. Unfair tournaments are characterized by a lead of one agent, which is modelled via a positive constant within the agent's production function. This function is given by the sum of individual effort and an idiosyncratic noise term, hence, they consider exactly the same tournament model as described in the first paragraph of Section 2. The

---

[10]Technically, this can be seen from (5) and (8).

[11]See also Bull et al. on "uneven" tournaments.

12

experimental results show that both types of players (i.e. the underdog i and the favorite j) significantly oversupply effort. Our results on the impact of emotions on incentives may help to explain the puzzling experimental findings: if emotions are relevant when trying to win a tournament and the primary incentive effects are not outweighed by possibly countervailing spillovers then emotional participants will choose significantly higher effort levels than non-emotional participants.

Finally, inspection of the first-order conditions (3) and (4) shows that the impact of the ability difference Δa (as a measure of agents' heterogeneity) on incentives is ambivalent. Let us, for the moment, neglect the existing interdependencies between equations (3) and (4) and focus on the single first-order conditions in isolation. On the one hand, Δa determines emotions that then determine perceived prizes and, hence, an agent's utility spread. Let us call this effect of Δa the *emotion effect*. If, for example, agents do not feel strong emotions when losing (winning) against more (less) able opponents as this outcome is the most likely one, then the only emotions that matter arise when strong contestants (i.e. favorites) lose against less able ones and when low able agents (i.e. underdogs) win against predominant opponents. Under this scenario, it is plausible to assume that in the former case the favorite's disappointment increases and hence his perceived loser prize decreases, but in the latter case the underdog's pride or joy and hence his perceived winner prize increase in the ability difference Δa. Under these conditions, we unambiguously have a positive emotion effect: Δa increases both types of emotions and, therefore, the agents' utility spreads, resulting in overall higher incentives.

On the other hand, the ability difference Δa appears in the density $f(\cdot)$, which leads to a kind of *competition effect*: Since this density is unimodal

13

with mode zero both equilibrium efforts will be small if $|e_i - e_j - \Delta a|$ is large.[12] If, in the initial situation, agent i has chosen more effort than agent j because $\Delta u_i\left(w_i^+, w_i^-\right) > \Delta u_j\left(w_j^+, w_j^-\right)$ and this effort level is so high that $e_i^* > e_j^* + \Delta a$, then a marginal increase in Δa will make the uneven competition less uneven, implying higher efforts for both agents. However, if initially $e_i^* < e_j^* + \Delta a$, then marginally increasing the ability difference will make the uneven situation more uneven, resulting in reduced efforts of both agents.

Now we take into account that the agents' effort choices are interrelated due to the tournament game. Implicitly differentiating the system of equations (3) and (4) yields

$$\frac{\partial e_i^*}{\partial \Delta a} = \frac{1}{|J|}\left(-\frac{\partial \Delta u_i}{\partial \Delta a}\bar{f}\underbrace{\left[-\Delta u_j \bar{f}' - c''(e_j^*)\right]}_{< 0 \text{ due to } SOC_j} - \Delta u_i \bar{f}'\left[c''(e_j^*) + \frac{\partial \Delta u_j}{\partial \Delta a}\bar{f}\right]\right)$$
(11)

$$\frac{\partial e_j^*}{\partial \Delta a} = \frac{1}{|J|}\left(-\frac{\partial \Delta u_j}{\partial \Delta a}\bar{f}\underbrace{\left[\Delta u_i \bar{f}' - c''(e_i^*)\right]}_{< 0 \text{ due to } SOC_i} - \Delta u_j \bar{f}'\left[c''(e_i^*) - \frac{\partial \Delta u_i}{\partial \Delta a}\bar{f}\right]\right).$$ (12)

In both (11) and (12), the expression in parentheses consists of two terms. As has been motivated in the next-to-last paragraph, let $\frac{\partial \Delta u_i}{\partial \Delta a} > 0$ and $\frac{\partial \Delta u_j}{\partial \Delta a} > 0$ due to a positive emotion effect. Then the first term is always positive in each derivative. However, the sign of the second term is ambiguous because of the ambiguity of the competition effect. Only if $\bar{f}' < 0$ we will have the clear-cut result $\frac{\partial e_i^*}{\partial \Delta a} > 0$. In the initial situation, we are at the RHS of the density $f(\cdot)$ with $e_i^* > e_j^* + \Delta a$. If now heterogeneity marginally increases, we get back to the peak of the density, which enhances incentives according to the competition effect so that both emotion effect and competition effect work into the same direction for agent i.[13] In this scenario, we obtain the

[12]In this case, we move to the tails of the density.

[13]Note that, according to Proposition 1, we have $\frac{\partial e_i^*}{\partial \eta_i} > 0$ and $\frac{\partial e_i^*}{\partial \eta_j} > 0$ at the RHS.

14

interesting result that the underdog i will exert higher effort if favorite j's ability and, hence, the heterogeneity measure Δa increases. However, we do not have the same clear-cut result for agent j because of the existing spillover effects.

3.2 Optimal Tournament Prizes and Emotions

Now we consider the principal's optimization problem at stage 1. Before we start note that the principal will usually not implement first-best effort levels that are characterized by $e^{FB} = \arg\max \{e_i + a_i + \varepsilon_i - c(e_i)\} \Rightarrow c'\left(e^{FB}\right) = 1$ $(i = 1, 2)$ since inducing incentives leads to risk costs due to the agents' risk aversion, and the principal has to compensate the agents for bearing risk according to their participation constraints. The principal chooses w_L and w_H in order to maximize his expected profits

$$\pi = e_i^* + e_j^* + a_i + a_j + 2E[\varepsilon] - w_L - w_H \tag{13}$$

subject to the agents' incentive constraints (3) and (4) and the two participation constraints

$$u_i\left(w_i^-\right) + \Delta u_i\left(w_i^+, w_i^-\right) F(e_i^* - e_j^* - \Delta a) - c(e_i^*) \geq \bar{u} \tag{14}$$

and

$$u_j\left(w_j^-\right) + \Delta u_j\left(w_j^+, w_j^-\right)\left(1 - F(e_i^* - e_j^* - \Delta a)\right) - c\left(e_j^*\right) \geq \bar{u}. \tag{15}$$

We obtain the following result:

Proposition 2 *When endogenously choosing optimal tournament prizes, the principal may benefit from both positive and negative emotions.*

15

Proof. First, consider a situation without emotions. Since $\frac{\partial \Delta u_i \left(w_i^+, w_i^- \right)}{\partial w_L} = -u_i' \left(w_i^- \right) \cdot \frac{\partial w_i^-}{\partial w_L} < 0$ $(i = 1, 2)$, the agents' incentives as well as the principal's expected profits decrease in the loser prize w_L. Hence, the principal will optimally choose that value of w_L that makes the participation constraint of the agent with the lower value of his objective function (i.e. $\min \left\{ U_i \left(L_i, e_i^* \right), U_j \left(L_j, e_j^* \right) \right\}$) just bind. However, the agent with the higher value $\max \left\{ U_i \left(L_i, e_i^* \right), U_j \left(L_j, e_j^* \right) \right\}$ will earn a positive rent in terms of utility in the optimum. Let, for example, agent i be this individual, and let ρ_i denote this rent. Suppose that we have an equilibrium at the LHS of $f \left(\cdot \right)$ (i.e. $e_i^* < e_j^* + \Delta a$). If we now introduce (marginal) positive and negative emotions for agent i, $\eta_i^+ > 0$ and $\eta_i^- > 0$, efforts e_i^* and e_j^* will rise according to Proposition 1. Let the winner prize w_H be adjusted downwards so that (via $\Delta u_j \left(w_j^+, w_j^- \right)$) effort e_j^* remains the same as before without emotions. Then there may exist situations in which the impact of η_i^+ and η_i^- (via $\Delta u_i \left(w_i^+, w_i^- \right)$) dominates the influence of the reduced winner prize so that e_i^* is larger than in the former case without emotions. Note that the higher winning probability increases the value of i's objective function, but the additional effort leads to further costs according to $c \left(e_i^* \right)$. In addition, the negative emotions η_i^- reduce $u_i \left(w_i^- \right)$. However, if i's rent ρ_i is sufficiently large, the participation constraint (14) will still hold. ∎

From the principal's point of view, emotions are a double-edged sword. On the one hand, they can enhance agents' incentives, which is beneficial for him. On the other hand, higher effort leads to higher effort costs, and negative emotions directly imply a utility loss for the agents. Both have to be taken into account by the principal because of the agents' participation

16

constraints.[14] However, if these costs have to be borne by the agent who earns a positive rent, the principal can benefit from the extra incentives due to emotions without paying for them. The additional costs will only reduce the agent's rent. Note that the agents are not protected by limited liability in the sense that we only allow for non-negative tournament prizes (i.e. $w_L, w_H \geq 0$).[15] In case of limited liability, the principal's possibilities to extract rents from the agents by choosing an appropriate loser prize are further restricted so that in the optimum typically *both* agents earn positive rents. In this situation, the principal has even more room to benefit from positive as well as negative emotions.

For analyzing the interplay of emotions and risk aversion, we have to specify the agents' utility functions further. Let, for example, each agent i have a quadratic utility function

$$u_i\left(\tilde{w}_i\right) = \tilde{w}_i - r_i\tilde{w}_i^2 \tag{16}$$

with $r_i > 0$ indicating i's degree of risk aversion and $\tilde{w}_i < 1/\left(2r_i\right)$.[16] Agent i's objective function is then given by

$$\begin{aligned}
U_i\left(L_i, e_i\right) &= E\left[\tilde{w}_i\right] - r_i E\left[\tilde{w}_i^2\right] - c\left(e_i\right) \tag{17}\\
&= w_i^- + \Delta w_i p_i - r_i\left(\left(w_i^-\right)^2\left(1 - p_i\right) + \left(w_i^+\right)^2 p_i\right) - c\left(e_i\right)
\end{aligned}$$

---

[14]Of course, from the principal's perspective, positive emotions are better than negative ones since only negative emotions immediately lead to a utility loss via $u_i\left(w_i^-\right)$.

[15]See Kräkel (2004) for the case of limited liability.

[16]See, for example, Müller and Machina (1987), Mas-Colell et al. (1995, 209), Rubinstein (2006, 97). Note that the assumption of a quadratic utility function is at most as special as the assumption of CARA preferences combined with normally distributed noise that is frequently applied in principal-agent models (e.g., Fershtman et al. 2003a). As an alternative, Lazear and Rosen use first-order and second-order Taylor series expansions to derive approximate results. However, these approximations are not very precise for several utility functions (e.g. for the CARA case).

17

with $\Delta w_i := w_i^+ - w_i^-$. For agent j, we obtain

$$U_j\left(L_j, e_j\right) = w_j^- + \Delta w_j\left(1 - p_i\right)$$
$$- r_j\left(\left(w_j^-\right)^2 p_i + \left(w_j^+\right)^2\left(1 - p_i\right)\right) - c\left(e_j\right) \quad (18)$$

with $\Delta w_j := w_j^+ - w_j^-$. Each agent receives his perceived loser prize and earns the additional perceived prize spread, Δw_i or Δw_j, in case of winning the tournament. In addition, each agent has to bear his risk costs, $r_i\left(\left(w_i^-\right)^2\left(1 - p_i\right) + \left(w_i^+\right)^2 p_i\right)$ or $r_j\left(\left(w_j^-\right)^2 p_i + \left(w_j^+\right)^2\left(1 - p_i\right)\right)$, and his costs from exerting effort.

In analogy to the general case, an equilibrium in pure strategies is characterized by the first-order conditions:

$$\left(1 - r_i\left(w_i^+ + w_i^-\right)\right)\Delta w_i f\left(e_i^* - e_j^* - \Delta a\right) = c'\left(e_i^*\right) \quad (19)$$
$$\left(1 - r_j\left(w_j^+ + w_j^-\right)\right)\Delta w_j f\left(e_i^* - e_j^* - \Delta a\right) = c'\left(e_j^*\right). \quad (20)$$

Note that the technical assumption from above that guarantees a non-decreasing utility function, $\tilde{w}_i < 1/\left(2r_i\right)$ $(i = 1, 2)$, implies that $1 - r_i(w_i^+ + w_i^-) > 0$ and $1 - r_j(w_j^+ + w_j^-) > 0$. According to (19) and (20), in equilibrium each agent chooses an effort level that equates marginal revenues and marginal costs. Moreover, the higher the risk coefficient r_i (or r_j, respectively), the lower will be the equilibrium effort of an agent.[17] The intuition for the impact of risk aversion on incentives can be seen from the agents' objective functions: risk costs are increasing in an agent's winning probability, therefore, weakening incentives. Altogether, the principal wants to maximize expected profits given by (13) subject to the incentive constraints (19) and (20) and

---

[17]Lazear and Rosen show that equilibrium efforts are also decreasing in an agent's risk aversion if the principal endogenously chooses optimal tournament prizes.

the participation constraints

$$w_i^- + \Delta w_i F\left(e_i^* - e_j^* - \Delta a\right) - c\left(e_i^*\right) \qquad (21)$$
$$-r_i \left(\left(w_i^-\right)^2 \left(1 - F\left(e_i^* - e_j^* - \Delta a\right)\right) + \left(w_i^+\right)^2 F\left(e_i^* - e_j^* - \Delta a\right)\right) \geq \bar{u}$$

and

$$w_j^- + \Delta w_j \left(1 - F\left(e_i^* - e_j^* - \Delta a\right)\right) - c\left(e_j^*\right) \qquad (22)$$
$$-r_j \left(\left(w_j^-\right)^2 F\left(e_i^* - e_j^* - \Delta a\right) + \left(w_j^+\right)^2 \left(1 - F\left(e_i^* - e_j^* - \Delta a\right)\right)\right) \geq \bar{u}.$$

Proposition 2 points out that positive and negative emotions can be beneficial for the principal since they generate extra incentives that may be free for the principal. The constraints (19)–(22) show that each agent's risk coefficient has two effects on this benefit. First, according to the incentive constraints (19) and (20), extra incentives due to emotions are smaller the larger r_i and r_j, respectively, are. This effect clearly reduces the potential benefits of emotions. Second, according to the participation constraints (21) and (22), the extra incentives also influence risk costs. On the one hand, positive emotions lead to large values of w_i^+ and w_j^+, which imply high risk costs. In this manner, the beneficial effects of emotions on incentives are further reduced. On the other hand, negative emotions are associated with small values of w_i^- and w_j^-, which decrease risk costs in (21) and (22). However, note that emotions lead to a larger perceived prize spread and, hence, to a larger income risk for the agents. Thus, in a more general setting overall risk costs should unambiguously increase by emotions. The findings can be summarized as follows:

Corollary 2 *If the agents are not too risk averse, the principal may benefit from both positive and negative emotions when endogenously choosing optimal tournament prizes.*

19

4   Discussion

In the given setting, the principal knows the agents' objective functions so that he can optimally design incentives by choosing appropriate tournament prizes. In particular, it is assumed that the principal observes the agents' abilities and knows their emotional propensities. Although this assumption is often used in principal-agent models, it does not always hold in practice. If principal and agents know each other for a long time (e.g. as an employer and his permanent staff), the simplifying common-knowledge assumption may be justified in principle. However, if principal and agents meet for the first time, the assumption will usually not hold. In the scenario analyzed in this paper, the principal can make use of his knowledge to design the optimal composition of the tournament if he has the choice between several heterogeneous agents.[18] For example, if emotions are strongest in asymmetric tournaments between a clear favorite and a clear underdog and the principal benefits from extra incentives due to emotions, he will strictly prefer such unfair tournaments to fair competitions with equally able opponents. If, however, the competition effect works against the emotion effect (see Subsection 3.1) and/or the agents are very risk averse so that emotional agents are not attractive for the principal, he will rather prefer even contests. Of course, in situations in which the principal is not aware of the agents' objective functions, this optimal seeding is impossible. Then the principal has to form beliefs about the agents' types. If there is asymmetric information, the principal may use a revelation mechanism to elicit private information from the agents. Perhaps the tournament itself can serve as a self-selection device since strictly emotional agents who are very risk averse will reject the offered tournament contract.

---

[18]For optimal seeding in tournaments, see Groh et al. (2003).

If the agents do not know their opponents' preferences, they have to rely on their respective beliefs when choosing effort at the tournament stage. Particularly, agents may have different desires to win the tournament but do not mutually know these preferences. Imagine that in this situation agents are equally strong so that emotions cannot arise from heterogeneity in ability. However, it is possible that agent A chooses a very large effort level because he has strong preferences for winning against agent B. If then B loses and learns after the tournament that he has underrated A's desire to win and, therefore, chosen a suboptimally small effort level, B may feel negative emotions such as anger because of his failure. If, in addition, agents do not have identical abilities and B is a clear favorite but loses, his anger will be even much stronger. Note that we might also have the opposite case where agent B overrates A's desire to win and, hence, chooses a very large effort level which leads to high effort costs. If afterwards B finds out that he made a mistake he might feel anger from having chosen excessively high effort. In both cases, emotions can simply emerge because of having learned about own mistakes and not because agents differ in ability.[19]

5 Conclusion

In this paper, we introduce a concept of emotions into the theory of rank-order tournaments. We analyze the impact of emotions on the agents' incentives and the principal's expected profit. It can be shown that the net effect

---

[19]There are parallels to the loser's curse and the winner's curse, both being discussed in the literature on common value auctions (see, e.g., Kagel and Levin 1986, Holt and Sherman 1994). However, in the common value auction, bidders make a mistake when estimating the true but unknown value of a commodity and not the unknown preferences of opponents.

21

of positive and negative emotions on both agents' efforts may be positive. Furthermore, the principal will benefit from emotional incentives if he need not directly pay for the enhanced incentives, that is if tournament prizes are exogenous, or if they are endogenous but the extra incentives due to emotions only reduce an agent's positive rent.

The concept of emotions used in this paper has a special focus. Here, we have concentrated on emotions that emerge when comparing one's own performance with the performance of other agents who participate in the same contest. By this, the interplay of emotions and incentives can be analyzed in detail. Moreover, if the principal is able to identify an agent's type and choose among heterogeneous agents, results can be derived concerning the optimal composition of tournaments from the principal's viewpoint. Finally, the concept is used in order to explain experimental findings on the oversupply of effort in tournaments that contradict standard economic theory.

The analysis of emotions can be extended in several directions. For example, this paper considers the impact of emotions on incentives. Perhaps there are also matching effects concerning different types of agents with different emotional attitudes. Considering such weak factors like the "chemistry" between co-workers may be important when deciding about the composition of departments and work groups. As another example, it may be interesting to discuss emotions in a dynamic setting. Over time there may be reinforcement effects concerning emotions such as disappointment or frustration and, hence, the existence of certain threshold levels may be decisive for agents' actions. Furthermore, in a dynamic context evolutionary aspects concerning the emergence or disappearance of certain emotional attitudes in work groups can be analyzed.

22

References

Baker, G.P., Gibbs, M., Holmström, B., 1994. The wage policy of a firm. Quarterly Journal of Economics 109, 921-955.

Bognanno, M.L., 2001. Corporate tournaments. Journal of Labor Economics 19, 290-315.

Bosman, R., van Winden, F., 2002. Emotional hazard in a power-to-take experiment. Economic Journal 112, 147-169.

Boyle, M., 2001. Performance reviews: perilous curves ahead. Fortune, May 28, 187-188.

Bull, C., Schotter, A., Weigelt, K., 1987. Tournaments and piece rates: an experimental study. Journal of Political Economy 95, 1-33.

Elster, J., 1998. Emotions and economic theory. Journal of Economic Literature 36, 47-74.

Eriksson, T., 1999. Executive compensation and tournament theory: empirical tests on Danish data. Journal of Labor Economics 17, 262-280.

Fershtman, C., Hvide, H.K., Weiss, Y., 2003a. A behavioral explanation of the relative performance evaluation puzzle. Annales d'Économie et de Statistique 71/72, 349-361.

Fershtman, C., Hvide, H.K., Weiss, Y., 2003b. Cultural diversity, status concerns and the organization of work. Working Paper No. 4-2003, Foerder Institute for Economic Research and Sackler Institute of Economic Studies.

23

Frank, R.H., Cook, P.J., 1996. The Winner-Take-All Society. New York: Penguin Books.

Green, J.R., Stokey, N.L., 1983. A comparison of tournaments and contracts. Journal of Political Economy 91, 349-364.

Groh, C., Moldovanu, B., Sela, A., Sunde, U., 2003. Optimal seedings in elimination tournaments. Discussion Paper, University of Bonn.

Hirshleifer, J., 1987. On the emotions as guarantors of threats and promises. In: Dupre, J. (Ed.). The Latest on the Best. Cambridge (MA): MIT Press, 307-326.

Höffler, F., Sliwka, D., 2003. Do new brooms sweep clean? – when and why dismissing a manager increases the subordinates' performance. European Economic Review 47, 877-890.

Holmström, B., 1999. Managerial incentive problems: a dynamic perspective. Review of Economic Studies 66, 169-182.

Holt, C.A., Sherman, R., 1994. The loser's curse. American Economic Review 84, 642-652.

Kagel, J.H., Levin, D., 1986. The winner's curse and public information in common value auctions. American Economic Review 76, 894-920.

Kahneman, D., Tversky, A., 1979. Prospect theory: an analysis of decision under risk. Econometrica 47, 263-291.

Kandel, E., Lazear, E.P., 1992. Peer pressure and partnerships. Journal of Political Economy 100, 801-817.

24

Knoeber, C.R., Thurman, W.N., 1994. Testing the theory of tournaments: an empirical analysis of broiler production. Journal of Labor Economics 12, 155-179.

Kräkel, M., 2004. Emotions and incentives. Bonn Econ Discussion Paper No. 14/2004.

Lazear, E.P., Rosen, S., 1981. Rank-order tournaments as optimum labor contracts. Journal of Political Economy 89, 841-864.

Loewenstein, G., 2000. Emotions in economic theory and economic behavior. American Economic Review, Papers and Proceedings 90, 426-432.

Machina, M.J., 1987. Choice under uncertainty: problems solved and unsolved. Journal of Economic Perspectives 1:1, 121-154.

Malcomson, J.M., 1984. Work incentives, hierarchy, and internal labor markets. Journal of Political Economy 92, 486-507.

Mantrala, M.K., Krafft, M., Weitz, B., 2000. An empirical examination of economic rationales for companies' use of sales contests. GEABA Discussion Paper # DP 00-07. German Economic Association of Business Administration.

Mas-Colell, A., Whinston, M.D., Green, J.R., 1995. Microeconomic Theory. New York: Oxford University Press.

Meyer, M.A., Vickers, J., 1997. Performance comparisons and dynamic incentives. Journal of Political Economy 105, 547-581.

Moldovanu, B., Sela, A., Shi, X., 2007. Contests for status. Journal of Political Economy 115, 338-363.

25

Müller, S.M., Machina, M.J., 1987. Moment preferences and polynomial utility. Economics Letters 23, 349-353.

Mui, V.-L., 1995. The economics of envy. Journal of Economic Behavior and Organization 26, 311-336.

Murphy, K.J., 1992. Performance measurement and appraisal: motivating managers to identify and reward performance. In: Bruns, W.J. Jr. (Ed.). Performance Measurement, Evaluation, and Incentives. Boston: Harvard Business School Press, 37-62.

Nalebuff, B.J., Stiglitz, J.E., 1983. Prizes and incentives: towards a general theory of compensation and competition. Bell Journal of Economics 14, 21-43.

O'Keeffe, M., Viscusi, W.K., Zeckhauser, R.J., 1984. Economic contests: comparative reward schemes. Journal of Labor Economics 2, 27-56.

Rosen, S., 1986. Prizes and incentives in elimination tournaments. American Economic Review 74, 701-715.

Rubinstein, A., 2006. Lecture Notes in Microeconomic Theory. Princeton and Oxford: Princeton University Press.

Schotter, A., Weigelt, K., 1992. Asymmetric tournaments, equal opportunity laws, and affirmative action: some experimental results. Quarterly Journal of Economics 107, 511-539.

Weigelt, K., Dukerich, J., Schotter, A., 1989. Reactions to discrimination in an incentive pay compensation scheme: a game-theoretic approach. Organizational Behavior and Human Decision Processes 44, 26-44.

26