

Modeling regional house prices

Dijk, Bram van; Franses, Philip Hans; Paap, Richard; Dijk, Dick van

Postprint / Postprint

Zeitschriftenartikel / journal article

Zur Verfügung gestellt in Kooperation mit / provided in cooperation with:

www.peerproject.eu

Empfohlene Zitierung / Suggested Citation:

Dijk, B. v., Franses, P. H., Paap, R., & Dijk, D. v. (2009). Modeling regional house prices. *Applied Economics*, 1-36.
<https://doi.org/10.1080/00036840903085089>

Nutzungsbedingungen:

Dieser Text wird unter dem "PEER Licence Agreement zur Verfügung" gestellt. Nähere Auskünfte zum PEER-Projekt finden Sie hier: <http://www.peerproject.eu> Gewährt wird ein nicht exklusives, nicht übertragbares, persönliches und beschränktes Recht auf Nutzung dieses Dokuments. Dieses Dokument ist ausschließlich für den persönlichen, nicht-kommerziellen Gebrauch bestimmt. Auf sämtlichen Kopien dieses Dokuments müssen alle Urheberrechtshinweise und sonstigen Hinweise auf gesetzlichen Schutz beibehalten werden. Sie dürfen dieses Dokument nicht in irgendeiner Weise abändern, noch dürfen Sie dieses Dokument für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, aufführen, vertreiben oder anderweitig nutzen.

Mit der Verwendung dieses Dokuments erkennen Sie die Nutzungsbedingungen an.

gesis
Leibniz-Institut
für Sozialwissenschaften

Terms of use:

This document is made available under the "PEER Licence Agreement". For more information regarding the PEER-project see: <http://www.peerproject.eu> This document is solely intended for your personal, non-commercial use. All of the copies of this documents must retain all copyright information and other information regarding legal protection. You are not allowed to alter this document in any way, to copy it for public or commercial purposes, to exhibit the document in public, to perform, distribute or otherwise use the document in public.

By using this particular document, you accept the above-stated conditions of use.

Mitglied der

Leibniz-Gemeinschaft



Modeling Regional House Prices

Journal:	<i>Applied Economics</i>
Manuscript ID:	APE-2009-0338
Journal Selection:	Applied Economics
Date Submitted by the Author:	28-May-2009
Complete List of Authors:	van Dijk, Bram; Erasmus University Rotterdam, Econometric Institute Franses, Philip Hans; Erasmus University Rotterdam, Econometric Institute Paap, Richard; Erasmus University Rotterdam, Econometric Institute van Dijk, Dick; Erasmus University Rotterdam, Econometric Institute
JEL Code:	C21 - Cross-Sectional Models Spatial Models < C2 - Econometric Methods: Single Equation Models < C - Mathematical and Quantitative Methods, C23 - Models with Panel Data < C2 - Econometric Methods: Single Equation Models < C - Mathematical and Quantitative Methods, C53 - Forecasting and Other Model Applications < C5 - Econometric Modeling < C - Mathematical and Quantitative Methods
Keywords:	cross-section dependence, cointegration, ripple effect

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60



For Peer Review

Modeling Regional House Prices*

Bram van Dijk Philip Hans Franses Richard Paap
Dick van Dijk[†]

Econometric Institute
Tinbergen Institute
Erasmus University Rotterdam

May 28, 2009

Abstract

We develop a panel model for regional house prices, for which both the cross-section and the time series dimension is large. The model allows for stochastic trends, cointegration, cross-equation correlations, and, most importantly, latent-class clustering of regions. Class membership is fully data-driven and based on the average growth rates of house prices, and the relationship of house prices with economic growth. We apply the model to quarterly data for the Netherlands. The results suggest that there is convincing evidence for the existence of two distinct clusters of regions, with pronounced differences in house price dynamics.

Keywords: cross-section dependence, cointegration, ripple effect

JEL Classification: C21, C23, C53

*We thank participants at the *Applied Economics* Ruby Anniversary Conference (Cambridge, September 2008) for useful comments, and René Segers for assistance with the graphs.

[†]Address for correspondence: Econometric Institute, Erasmus University Rotterdam, P.O. Box 1738, 3000 DR Rotterdam, The Netherlands, e-mail: djvandijk@ese.eur.nl.

1 Introduction

Real estate prices in many countries have experienced a dramatic boom in recent years (IMF, 2004). At the same time, the extent of the price increase appears to vary substantially across different regions within a given country. In the Netherlands, for example, it is commonly believed that house prices in *Amsterdam* and the densely populated western part of the country have increased far more than prices in the smaller cities and rural areas in the east. As house prices are typically available per region or city, we may analyze these data at such a disaggregate level, to examine whether indeed regions or cities behave differently, perhaps in terms of trends, but also in terms of response to outside economic shocks. In this paper we develop a time series model that suits this purpose.

Most regional house prices have the following properties. First, they tend to display a trend, and historical price patterns suggest that this trend probably is not deterministic but stochastic. In particular, house prices show ‘bubble’-type behavior, where prolonged periods of steady increases of the price level suddenly end with a sharp drop followed by a period of low price levels, suggesting that trends are unlikely to be deterministic. Second, for different regions within a country these stochastic trends should somehow be linked. It is not plausible that prices in different regions would diverge indefinitely or that certain regions would not respond to common macroeconomic shocks. So, a model for regional house prices should allow for some form of common trends. Third, it can be expected that adjacent regions show similar price patterns, although this may also be the case for regions far apart geographically but with similar economic or demographic characteristics. Hence, a suitable model should allow for similarities in the dynamic behavior of house prices across regions. An intuitively appealing possibility is to consider a model that allows for groups or clusters of regions, where house price dynamics in regions within a given cluster are the same, while they are different across clusters. Preferably, such a model should not require ex-ante or exogenous assignment of regions to specific clusters.

1
2
3
4
5
6 In fact it would be best if the data themselves were allowed to indicate if clusters exist
7 and if so, which regions belong to which cluster.
8
9

10 In this paper we extend the latent-class panel time series model introduced by Paap
11 *et al.* (2005) to capture these different properties of regional house prices. The key feature
12 of this model is that the clustering of regions is purely data-driven, where cluster mem-
13 bership is based on characteristics corresponding to two specific research questions we
14 want to address. The first question is whether prices in all regions have the same average
15 growth rate. Note that a common trend specification across the regions entails that their
16 growth rates must be somehow compatible, but it still leaves open the possibility that
17 house prices in some regions grow faster than in others. The second question we consider
18 is the way the house prices in each region react to changes in the overall economic situ-
19 ation, which we measure by GDP. We examine both the size of the effect from GDP on
20 the house prices and the speed at which regions react to changes in GDP.
21
22
23
24
25
26
27
28
29
30
31
32

33 We apply our model to house price data for the Netherlands, comprising 76 regions
34 for which we have quarterly data for the period 1985Q1-2005Q4. We find that the 76
35 regions can be grouped into two clusters. The first cluster consists mainly of regions in
36 the east of the country. These are mainly rural areas that are close to the larger cities,
37 especially close to the *Randstad* (consisting of *Utrecht*, *Amsterdam*, *Den Haag*, *Rotterdam*
38 and other cities in the area). This cluster reacts both stronger and faster to changes in
39 GDP. The average growth rate does not vary over the regions.
40
41
42
43
44
45
46

47 There are not many studies that describe regional house prices. Cameron *et al.* (2006)
48 build a model from inverse demand equations. They have, however, only a limited number
49 (9) of regions, and their model would not work in our situation where we have many more
50 (76) regions, as we will describe below. Malpezzi (1999) constructs an error correction
51 model for regional house prices. The parameters of this model are however not allowed
52 to vary across regions. Holly *et al.* (2008) model US house prices at the state level. Their
53 model is 'fully heterogenous' in the sense that it has different parameters for each region.
54
55
56
57
58
59
60

1
2
3
4
5
6 In this paper we cover the middle ground, that is, the model parameters are allowed to
7 vary across *groups* of regions but not across each region individually.
8
9

10 Before we propose our latent-class model, we first provide some details on the house
11 price data in Section 2. We consider two decades of quarterly house prices on 76 regions in
12 the Netherlands. We discuss their trending behavior by performing panel unit root tests
13 and we also show that the growth rates in different regions show strong cross-correlations.
14 Using multidimensional scaling techniques we get a first impression if and how these 76
15 regions could get clustered. Then, in Section 3, we put forward our model specification,
16 highlighting the underlying data-driven clustering mechanism. In addition, we describe
17 the method used for parameter estimation. In Section 4 we first present our estimation
18 results, and give interpretation to the various outcomes. Next, we take a look at impulse
19 response functions of the house prices with respect to a shock in GDP and in the interest
20 rate. In Section 5 we conclude with some limitations and we outline topics for further
21 research.
22
23
24
25
26
27
28
29
30
31
32
33

34 35 36 37 **2 Data** 38 39

40 The Dutch real estate agent association [NVM] publishes quarterly data on house prices
41 for $N = 76$ regions in the Netherlands. Our dataset covers the sample period 1985Q1-
42 2005Q4 ($T = 84$ quarters). Hence, we have a panel database where both the cross-section
43 dimension N and the time dimension T are fairly large.
44
45
46
47

48 The way the country is divided into 76 regions is determined by the NVM. Macroeconomic
49 data, such as output and inflation, are not available for this particular specification
50 of regions. Other (macro) variables that we use in our model are therefore measured at
51 the country level. In particular, this concerns the interest rate (obtained from the Dutch
52 Central Bank) and quarterly real GDP (from Statistics Netherlands). The GDP series
53 is available until 2005Q2. We obtain real house prices by deflating with the consumer
54
55
56
57
58
59
60

1
2
3
4
5
6 price index [CPI] (from Statistics Netherlands). In addition, we seasonally adjust the real
7
8 GDP series using the Census X-12 algorithm (available in EViews 5.1). We denote the
9
10 real house price in region i at time t as $p_{i,t}$, and real GDP as y_t .
11

12
13 [Figure 1 about here.]
14
15

16
17 Figure 1 shows time series of $\log(p_{i,t})$ for three specific regions: *Noordwest-Friesland*,
18 which usually is the least expensive region, *Bunnik/Zeist*, which usually is the most ex-
19 pensive region, and *Amsterdam*, which is in between. On top we also plot $\log(y_t)$ (scaled
20 to limit the size of the vertical axis in the graph). Comparing the graphs in Figure 1
21 suggests that real house prices increase slightly faster than real GDP. Prices in *Bun-*
22 *nik/Zeist* and *Amsterdam* show substantial variations in the trend growth rate over time,
23 with alternating periods of steep price increases and of stable or falling prices. Especially
24 the ‘hump’ in the prices around 2000 stands out clearly. This suggests that the trend
25 in the house prices is stochastic rather than deterministic. Furthermore, as the trending
26 behavior of the different price series seems quite similar regional house prices may well be
27 cointegrated.
28
29
30
31
32
33
34
35
36
37
38
39
40

41 2.1 Unit roots and cointegration

42
43 To test whether these visual impressions from Figure 1 can be given more formal statistical
44 support, we perform panel unit root tests on the regional house prices. Two of the most
45 popular tests in the literature are those from Levin *et al.* (2002) [LLC] and Im *et al.*
46 (2003) [IPS], see Breitung and Pesaran (2008). These tests have as null hypothesis the
47 presence of a unit root in all the series in the panel. The alternative hypotheses are
48 different however. Levin *et al.* (2002) assume that the house price dynamics are the same
49 for each region, and therefore the alternative hypothesis is that all regional house prices
50 are stationary. Im *et al.* (2003), however, have as alternative hypothesis that at least
51
52
53
54
55
56
57
58
59
60

1
2
3
4
5
6 one regional house price is stationary. Both these tests assume that there is no cross-
7 correlation between different series in the panel. In fact, they are not consistent if such a
8 dependency is present, which is quite likely in our case. Alternative tests that do allow
9 for cross-section dependence are available, like the one in Moon and Perron (2004), but
10 these usually rely on asymptotics that require T to be much larger than N , while in our
11 case they are about equal.
12
13
14
15
16
17

18 To meet our data characteristics, we therefore employ the cross-sectionally augmented
19 IPS [CIPS] test, recently developed in Pesaran (2007). This allows for cross-sectional
20 dependence, and is also valid when N is larger than T . The idea of the CIPS test is
21 to add the cross-section averages of the lagged levels and first differences to the familiar
22 augmented Dickey-Fuller [ADF] regression equation. If it can be assumed that the cross-
23 correlations are caused by a common factor, then this common factor must also be present
24 in the cross-section averages. Adding these to the ADF equations should then get rid of
25 the common factor in the residuals and thus correct for the presence of cross-correlations.
26
27
28
29
30
31
32
33

34 As the CIPS test is known to have reduced power relative to the IPS and LLC tests
35 in case cross-correlation is not present, we test whether we really should use the CIPS
36 test instead of these simpler tests. For this purpose we use the cross-section dependence
37 [CD] test of Pesaran (2004) and the adjusted LM [LM_{adj}] test of Pesaran *et al.* (2008).
38 These tests both use the cross-correlations between the residuals of the individual ADF
39 regressions for the different regions. The CD test takes a simple sum which is scaled such
40 that it has a standard normal distribution under the null hypothesis of no cross-sectional
41 dependence. Therefore, the CD test has little power in case there are both positive and
42 negative correlations such that the average is close to zero. The LM_{adj} test, however, is also
43 valid in this case as it employs the squares of the cross correlations in the construction of
44 the test statistic. However, the LM_{adj} test is less robust against non-normally distributed
45 error terms and exhibits size distortions, especially when N is much larger than T .
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60

[Table 1 about here.]

Table 1 gives the result of these tests for the panel of quarterly growth rates in house prices $\Delta \log(p_{i,t})$, where Δ denotes the first-difference filter, and of $\log(p_{i,t}) - \log(p_{34,t})$, which is the difference of each series with the log house prices in *Amsterdam* (region 34, see Appendix A). The number of lagged (first) differences is allowed to vary across each (C)ADF equation and is determined by minimizing BIC. Adding a lagged variable means losing one observation, therefore we actually minimize BIC/T , see Cameron and Trivedi (2005, pp. 279) or the definition of BIC given in Franses and Paap (2001). Each (C)ADF regression equation contains an intercept and a trend.

From the second column of Table 1 we see that for the first difference of the log house prices there is substantial cross-sectional dependence, according to both the CD and LM_{adj} tests. Next, we see that all three unit root tests reject the presence of a unit root in these growth rate series. Results for the difference between the log price in a region and the log price in *Amsterdam* (region 34) appear in the third column of Table 1. The reason for examining the log price differences with respect to *Amsterdam* is that finding these to be stationary, we can conclude that the house prices in each region are cointegrated. Again, the CD and LM_{adj} tests indicate that there is substantial cross-sectional dependence. Next, the LLC and IPS unit root tests do not reject the presence of a unit root, but the CIPS test does. Since the LLC and IPS tests are not valid in case of cross-sectional dependence, we rely on the CIPS test and conclude that the log house prices in each region are cointegrated. Note that the $(1, -1)$ cointegration relationships suggested by the results in Table 1 are quite plausible. It means that the difference between the log of house prices, or, equivalently the ratio of house prices, in each region is a stationary process. This constrains the long-term growth of house prices in each region to be about the same.

[Figure 2 about here.]

2.2 Clusters

[Figure 3 about here.]

Before we turn to our conditional clustering analysis using latent class techniques we consider unconditional clustering based on the correlations of the house price growth rates or of the residuals of the ADF regressions used above. For this purpose, we use multi-dimensional scaling [MDS], which results in the graphs shown in Figure 2 and 3.

Although the graphs in these figures are rather different, they basically lead to the same conclusion that there are no apparent clusters. Hence, dividing the regions into different groups based only on the cross-correlations of the regional house prices is not a meaningful possibility. Apparently, we need a more sophisticated clustering method, perhaps based on latent classes, as we will propose in the next section.

3 The model

In this section we put forward the specification of the latent-class panel time series model for describing the regional house prices. We first discuss the characteristics of the model, and then we outline the parameter estimation procedure.

3.1 Representation

Our starting point is the latent-class panel time series model developed by Paap *et al.* (2005). The crucial idea behind this model is that the individual time series may be grouped into a limited number of clusters. Within each cluster, a linear model is assumed to describe the dynamic behavior of the time series. The clusters are defined such that the model parameters are the same for all time series within a cluster, but they are different across clusters. Hence, this model covers the middle ground between a pooled regression model, where the model parameters are constrained to be the same for all regions, and

1
2
3
4
5
6 a ‘fully heterogenous’ model, where the parameters are allowed to be different for each
7 individual region. Whereas a pooled regression model may be too restrictive, a fully
8 heterogenous model may be too flexible and ignores the possible similarities between
9 regions. Finally, the key feature of the model of Paap *et al.* (2005) is that the number
10 of clusters in the model as well as the allocation of the individual time series to different
11 clusters is purely data-based. This avoids *ex ante*, and necessarily subjective, grouping
12 of regions according to geographical location or economic or demographic characteristics,
13 for example.
14
15
16
17
18
19
20
21

22 In our model for quarterly growth rates of house prices we allow for more flexibility
23 than was done in Paap *et al.* (2005). As mentioned, there are two research questions we
24 want to answer with our model and each question corresponds to different parameters that
25 can vary across the latent classes. The first is whether the mean growth rates of house
26 prices are the same across all regions. We therefore allow the clusters to have a different
27 average growth rate by allowing for a class-specific intercept. To facilitate interpretation,
28 we demean all other variables in the model such that the intercept is equal to the average
29 growth rate of the house prices in the regions in a cluster.
30
31
32
33
34
35
36
37
38

39 The second question we wish to answer with our model is whether the house prices in
40 regions follow the trend in real GDP. We add an error correction variable linking regional
41 real house prices and real GDP, where the long-run parameter should be estimated. This
42 long-run parameter determines the size of the effect of GDP on the house prices. The
43 adjustment parameter indicates how fast the house prices in a region react to changes in
44 GDP.
45
46
47
48
49
50

51 Based on the above discussion, we propose the following latent-class panel time series
52 model for regional house prices in the Netherlands
53
54
55
56

$$\Delta \log(p_{i,t}) = \beta_{0,k_i} + \beta_{1,k_i} [\log(p_{i,t-1}) + \gamma_{k_i} \log(y_{t-1})] + \eta_{i,t}. \quad (1)$$

1
2
3
4
5
6 The β and γ parameters are class-specific parameters, where the subscript $k_i = 1, \dots, K$
7 denotes the latent class which region i belongs to with K being the number of latent
8 classes. We denote the probability that a region belongs to latent class k , or the mixing
9 proportions, as π_k . Naturally it must hold that, $0 < \pi_k < 1$ and that $\sum_{k=1}^K \pi_k = 1$.
10
11
12
13

14 As the house prices of each regions are cointegrated with GDP, they are also cointe-
15 grated amongst themselves. This can easily be seen in the following way. Both $p_{i,t} - \gamma_{k_i} y_t$
16 and $p_{j,t} - \gamma_{k_j} y_t$ are stationary series. Now, consider the following expression,
17
18
19
20
21

$$(p_{i,t} - \gamma_{k_i} y_t) - \delta(p_{j,t} - \gamma_{k_j} y_t) = (p_{i,t} - \delta p_{j,t}) - (\gamma_{k_i} - \delta \gamma_{k_j}) y_t. \quad (2)$$

22 The LHS of (2) is stationary, therefore the RHS is also a stationary series. For $\delta = \gamma_{k_i} / \gamma_{k_j}$
23 the second term on the RHS of (2) will disappear, therefore regions i and j must have a
24 $(1, -\delta)$ cointegration relationship. Two regions in the same cluster will therefore have a
25 $(1, -1)$ cointegration relationship, because they share the same γ parameter. As we have
26 seen in Section 2.1, there is support for exactly this relationship.
27
28
29
30
31
32
33
34

35 Even though model (1) includes $\log(y_{t-1})$, which is the same for all regions, there
36 may still be some cross-section correlation among the house prices that is not captured.
37 Therefore, following Holly *et al.* (2008), we allow the error term $\eta_{i,t}$ in (1) to be correlated
38 across regions, but assume that this correlation is due to dependence on certain common
39 factors. To be precise, we consider the specification
40
41
42
43
44
45
46
47

$$\eta_{i,t} = \alpha_{1,i} \Delta \log(y_{t-1}) + \alpha_{2,i} I_{t-1} + \alpha_{3,i} \Delta \log(p_{t-1}) + \varepsilon_{i,t}, \quad (3)$$

48 where I_{t-1} denotes the interest rate at time $t - 1$, p_{t-1} denotes the average house price in
49 the Netherlands at time $t - 1$ and where $\alpha_{l,i}$ for $l = 1, 2, 3$ are region-specific parameters.
50 The residuals $\varepsilon_{i,t}$ are now assumed to be independently normally distributed with a region-
51 specific variance σ_i^2 .
52
53
54
55
56
57
58

59 In the application below, we demean all variables in (1) and (3) and hence the inter-
60

cepts β_{0,k_i} in (1) are equal to the average growth rates of the house prices in the latent classes k_i for $k_i = 1, \dots, K$.

3.2 Estimation

The parameters in our model (1) with (3) can be estimated as outlined in Paap *et al.* (2005), using the EM algorithm of Dempster *et al.* (1977). This makes use of the full data log-likelihood function, that is, the joint density of the house prices and the latent classes k_i , which we specify in detail below. The EM algorithm is an iterative maximization algorithm, which alternates between two steps until convergence occurs. In the first step (E-step) we compute the expected value of the full data log-likelihood function with respect to the latent classes k_i , $i = 1, \dots, N$, given the house prices and the current values of the model parameters. In the second step (M-step) we maximize the expected value of the full data log-likelihood function with respect to the model parameters. As the model given the class memberships can be written as a standard linear regression, the M-step amounts to a series of (weighted) regressions. As the EM algorithm maximizes the log-likelihood function, the resulting estimates of the model parameters are equal to the maximum likelihood [ML] estimates. We can therefore compute standard errors of the estimates using the second derivative of the log-likelihood function.

Note that due to the presence of the term $\beta_{1,k_i}[\log(p_{i,t-1}) + \gamma_{k_i} \log(y_{t-1})]$ the model in (1) is actually nonlinear in the parameters. To deal with this issue, we follow Boswijk (1994) and rewrite the model as

$$\Delta \log(p_{i,t}) = \beta_{0,k_i} + \beta_{1,k_i} \log(p_{i,t-1}) + \beta_{2,k_i} \log(y_{t-1}) + \eta_{i,t}, \quad (4)$$

where $\beta_{2,k_i} = \beta_{1,k_i} \gamma_{k_i}$. Note that (4) is linear in the parameters, which facilitates estimation. The ML estimate $\hat{\gamma}_{k_i}$ can be obtained from the ML estimates of β_{1,k_i} and β_{2,k_i} as $\hat{\beta}_{2,k_i} / \hat{\beta}_{1,k_i}$.

The full data likelihood function, that is, the joint density of $\mathcal{P} = \{\{\Delta \log p_{i,t}\}_{t=1}^T\}_{i=1}^N$ and $\mathcal{K} = \{k_i\}_{i=1}^N$ is given by

$$l(\mathcal{P}, \mathcal{K}; \theta) = \prod_{i=1}^N \left(\prod_{k=1}^K \left(\pi_k \prod_{t=1}^T \frac{1}{\sigma_i} \phi(\varepsilon_{i,t}^k / \sigma_i) \right)^{I[k_i=k]} \right), \quad (5)$$

where $\phi(\cdot)$ denotes the probability density function of a standard normal random variable and θ is a vector containing all model parameters. The error term at time t for region i belonging to cluster k is defined as

$$\varepsilon_{i,t}^k = \Delta \log p_{i,t} - x'_{i,t} \beta_k - w'_t \alpha_i, \quad (6)$$

where $x_{i,t}$ is the (3×1) vector with the regressors appearing in (4) and β_k contains the corresponding parameters for cluster k . Similarly, w_t is the (3×1) vector with common factors in the specification for $\eta_{i,t}$ in (3), and $\alpha_i = (\alpha_{1,i}, \alpha_{2,i}, \alpha_{3,i})'$ containing the parameters for region i .

The expectation of the full data log-likelihood function with respect to $\mathcal{K} | \mathcal{P}, \theta$ [E-step] is given by

$$\mathcal{L}(\mathcal{P}; \theta) = \sum_{i=1}^N \left(\sum_{k=1}^K \hat{\pi}_{i,k} \left(\ln \pi_k + \sum_{t=1}^T -\frac{1}{2} \ln \sigma_i^2 - \frac{1}{2} \ln 2\pi - \frac{(\varepsilon_{i,t}^k)^2}{2\sigma_i^2} \right) \right), \quad (7)$$

where $\hat{\pi}_{i,k}$ denotes the conditional probability that region i belongs to class k . This is equal to

$$\hat{\pi}_{i,k} = \frac{\pi_k \prod_{t=1}^T \frac{1}{\sigma_i} \phi(\varepsilon_{i,t}^k / \sigma_i)}{\sum_{l=1}^K \pi_l \prod_{t=1}^T \frac{1}{\sigma_i} \phi(\varepsilon_{i,t}^l / \sigma_i)}. \quad (8)$$

In the M-step, we need to maximize (7) with respect to the parameters β_k , π_k , $k = 1, \dots, K$ and α_i , σ_i^2 for $i = 1, \dots, N$. We perform this maximization step sequentially. First, we optimize over β_k keeping the other parameters fixed. This can be done by a simple weighted regression of $\Delta \log(p_{i,t}) - w'_t \alpha_i$ on $x_{i,t}$ with weights given by $\sqrt{\hat{\pi}_{i,k}} / \sigma_i$.

Clearly, we want regions with a larger probability of belonging to class k to have a larger weight in estimating β_k . At the same time, regions with a larger standard deviation of the error term σ_i should get a smaller weight, as their house prices contain relatively more noise and less information about β_k . Each β_k , $k = 1, \dots, K$ is estimated in a separate weighted regression.

Second, we optimize the log-likelihood function over α_i for $i = 1, \dots, N$. We do this by regressing $\sum_{k=1}^K \hat{\pi}_{i,k} [\Delta \log(p_{i,t}) - x_{i,t} \beta_k]$ on w_t . The dependent variable in this regression is the conditional expectation of $\eta_{i,t}$. We perform these regressions for each region separately.

Next, the new estimate of σ_i^2 is given by

$$\sigma_i^2 = \frac{1}{T} \sum_{t=1}^T \sum_{k=1}^K \hat{\pi}_{i,k} (\varepsilon_{i,t}^k)^2 \quad (9)$$

for $i = 1, \dots, N$. Finally, the mixing proportions are updated by averaging the conditional class membership probabilities, that is,

$$\pi_k = \frac{1}{N} \sum_{i=1}^N \hat{\pi}_{i,k} \quad (10)$$

for $k = 1, \dots, K$.

As we maximize over the parameters sequentially in the M-step, we do not reach the optimum of the expected full data log-likelihood function (7) in each iteration of the EM-algorithm. We can repeat the individual update steps until convergence, but this is not necessary. Indeed, Meng and Rubin (1993) have shown that an increase in the full-data log-likelihood function in the M-step is sufficient for the EM algorithm to converge to the maximum of the log-likelihood function.

Determining the appropriate number of latent classes is not straightforward. We cannot use a standard statistical test, due to the Davies (1977) problem of unidentified

1
2
3
4
5
6 nuisance parameters under the null hypothesis. The usual approach is using a criterion
7
8 function balancing the fit and the complexity of the model, where the model fit is mea-
9
10 sured by the value of the log-likelihood function while the number of model parameters
11
12 provides a measure of complexity. The most well-known criteria are the Akaike infor-
13
14 mation criterion [AIC] and the Bayesian information criterion [BIC]. Bozdogan (1994)
15
16 suggests that the AIC should have a penalty factor of 3 instead of 2 in the case of mixture
17
18 models. Indeed, Andrews and Currim (2003) show that this AIC-3 criterion outperforms
19
20 other criteria. Bozdogan (1987) modifies the AIC into the so-called consistent Akaike in-
21
22 formation criterion [CAIC], which is almost equal to BIC. He shows that when the sample
23
24 size is large the CAIC and BIC criteria perform better than AIC. We will consider all
25
26 four criteria below.
27
28
29
30

31 **4 Empirical results**

32
33
34 In this section we discuss the results of applying our model to the regional house price
35
36 data for the Netherlands described in Section 2. The effective sample period ranges from
37
38 1985Q3 (because we have $\Delta \log(p_{t-1}) = \log(p_{t-1}) - \log(p_{t-2})$ in our model) to 2005Q2
39
40 (because we only have real GDP data until 2005Q2), giving $T = 80$ data points in the time
41
42 series dimension. To obtain a first impression of the extent of similarities across regions,
43
44 we start by estimating a fully heterogenous model allowing for different parameters for
45
46 each region. Next, we provide estimation results for the model with a limited number
47
48 of latent classes. Finally, we consider impulse-response functions for three interesting
49
50 scenarios to provide further interpretation of the model.
51
52
53
54

55 **4.1 A fully heterogenous model**

56
57 We first estimate the parameters in a fully heterogenous model, that is, we estimate the
58
59 model in (1) with (3) allowing for different parameters for each individual region. This
60

1
2
3
4
5
6 essentially is a model with $K=76$ latent classes, in which case each region forms a separate
7
8 class.
9

10
11 [Table 2 about here.]
12

13
14 [Figure 4 about here.]
15
16

17
18 The mean, minimum and maximum of each parameter of the 76 regions can be found
19 in Table 2. Figure 4 displays the histograms for the 76 estimated values for each of the
20 parameters β_j , $j = 0, 1$, and γ in (1). The top panel shows the intercepts, β_0 , which
21 equal the quarterly growth rates. These are all positive, reflecting the upward trend in
22 the house prices, and range between 0.6% and 1.3% per quarter. The middle panel of
23 Figure 4 shows the results for the adjustment parameter for the cointegration term with
24 GDP. We find some positive values, which is not as expected, as these imply divergence
25 between GDP and the house prices in that region. Finally, the histogram in the bottom
26 panel shows the parameter γ in the cointegration relationship with GDP, which we expect
27 to be negative as we expect the house prices and GDP to move in the same direction.
28 Table 2 also shows the results for the α parameters from (3). Again we find that they
29 show some counterintuitive signs and a relatively large spread.
30
31
32
33
34
35
36
37
38
39
40
41

42 We can see from these results that some form of aggregation may be useful, as we now
43 get a wide variety of parameter estimates, with sometimes quite implausible results. At
44 the same time, this variety also suggests that we should perhaps better not restrict the
45 parameters to be the same across all regions. Hence, it may be optimal to allow for a
46 limited number of different clusters.
47
48
49
50
51
52

53 4.2 A model with latent classes

54
55 [Table 3 about here.]
56
57
58
59
60

1
2
3
4
5
6 A major issue for successful application of the latent-class panel time series model is of
7 course determining the appropriate number of latent classes. As discussed in Section 3.2,
8 we consider four different information criteria for this purpose. Table 3 shows the values
9 of these criteria for models with one to four and 76 classes. For all criteria, we see that
10 going from a homogenous model (with a single class) to two classes amounts to a relatively
11 large improvement in the balance of model fit and complexity. After this, adding more
12 classes does not improve any of the criteria. We therefore focus on the model with two
13 latent classes.
14
15
16
17
18
19
20
21

22 [Table 4 about here.]
23
24
25

26 The estimation results for the model with two latent classes are given in Table 4.
27 Additionally, Table 5 gives the results for a series of Wald tests which we use to examine
28 whether the parameters for the different classes are significantly different from each other.
29 The estimation results show that the regions in the two latent classes do indeed differ from
30 each other in several important respects. First, the estimated intercepts show that the
31 average growth rate in class 1 is slightly higher than in class 2¹. This difference is not
32 significant though, as can be seen from the second row of Table 5. The average growth
33 rate in class 1 is equal to 1.2% per quarter, or 4.8% annually, while the house prices in
34 class 2 grow with 1.1% per quarter, or 4.4% annually.
35
36
37
38
39
40
41
42
43
44

45 Second, examining the cointegration relationship with GDP, we find that class 1 has
46 a significantly larger adjustment parameter. Thus, the house prices in regions belonging
47 to cluster 1 react faster to changes in GDP than the house prices in class 2.
48
49
50

51 [Table 5 about here.]
52
53
54

55 Finally, The cointegration relationship between house prices and GDP itself, is also
56 significantly different across the classes. For class 1, it is (1, -1.89), meaning that in the
57

58 ¹Recall that we demeaned all other variables the model, so the intercepts represent the average growth
59 rates.
60

1
2
3
4
5
6 long run the house prices in the regions in this cluster grow almost twice as fast as GDP.
7
8 In class 2 the cointegration relationship is $(1, -1.68)$. These long term relationships may
9
10 not be very plausible, however, as we could already see from Figure 1, they are a good
11
12 description of the development of house prices and GDP in the sample period.
13
14

15
16 [Table 6 about here.]
17

18
19 As we showed in Section 3, the cointegration relationship of each region with GDP
20 entails that the regions are also cointegrated among themselves. The long term parameter
21 is only influenced by the γ parameters of the two regions involved, and thus only depends
22 on the class membership of the two regions. Table 6 shows these cointegration relation-
23 ships between the house prices of regions from any of the two clusters. First, we see that
24 two regions that belong to the same cluster are $(1, -1)$ cointegrated. This is actually very
25 intuitive, as they have follow the same trend relative to the trend of GDP, they must
26 follow the exact same trend themselves. Next we find that a region from cluster 1 is
27 $(1, -1.12)$ cointegrated with a region from cluster 2. This corresponds with the slightly
28 higher growth rate in class 1.
29
30
31
32
33
34
35
36
37
38

39 The parameters in (3) are region-specific, and full estimation results are not reported
40 to save space. Only 11% of the $\alpha_{1,i}$ parameters is significant, suggesting that the impact
41 of GDP on the house prices is mostly captured by the cointegration term. Moreover, only
42 22% has the expected positive sign. The $\alpha_{2,i}$ parameters are mostly negative, and only one
43 region has an (insignificant) positive value. Furthermore, for 63% of the regions the $\alpha_{2,i}$
44 parameter is significant at the 5% level, indicating that the interest rate indeed influences
45 the house prices in the expected direction. The $\alpha_{3,i}$ parameters, relating the growth of
46 the house price in a region to growth of the average house price in the Netherlands in the
47 previous quarter, is positive for 88% of the regions, but only significant for 42% of these
48 regions.
49
50
51
52
53
54
55
56
57
58
59
60

The latent classes

[Figure 5 about here.]

The parameter estimation results obviously become more interesting if we know which regions belong to each of the two classes. Therefore, we compute the conditional class membership probabilities using (8). The resulting classification of the regions is shown in Figure 5. Regions are colored based on $\hat{\pi}_{i,1}$, the probability of belonging to class 1. Regions are colored in four shades of grey. For the regions that are colored in the lightest shade it holds that $\hat{\pi}_{i,1} \leq 0.2$. For regions colored in subsequently darker shades of grey it holds that $0.2 < \hat{\pi}_{i,1} \leq 0.4$, $0.4 < \hat{\pi}_{i,1} \leq 0.6$, or $0.6 < \hat{\pi}_{i,1} \leq 0.8$. There were no regions with $\hat{\pi}_{i,1} > 0.8$. It can be seen that most regions are either very dark or very light, suggesting that the classification is very clear for most regions. In fact, the average value of $\max(\hat{\pi}_{i,1}, \hat{\pi}_{i,2})$ is equal to 0.83.

We find that class 1 contains mainly rural regions surrounding the big cities in the Netherlands. The regions in this class mainly cover parts of *Noord-Brabant* and the *Veluwe*. Even though the East belongs almost completely to class 1, the larger cities of the East, like *Zwolle*, *Almelo*, *Hengelo*, *Enschede*, and *Arnhem* are part of class 2.

Class 2 contains different types of regions. First, it contains many large cities in different parts of the country, like *Breda* and *Groningen*, as well as almost all of the regions in the *Randstad*, the densely populated western part of the country. At the same time some rural regions, like *Zeeland*, *Zuid-Limburg* and regions in the North belong to this class with high probability. Note that these rural regions are not as close to the *Randstad* as most of those in class 1.

A possible explanation for our results is the increased number of commuters that live in the regions belonging to class 1 and who work in the large western cities. If the number of commuters increases, it is likely that they move to regions in cluster 1, as these are still at traveling distance from the *Randstad*. This development has two consequences for the

1
2
3
4
5
6 regions in class 1. First, the average income in these regions is likely to increase, as the
7 individuals who move away from the cities are relatively wealthy. The second consequence
8 is an increase of housing quality in these regions, as wealthier people leaving the cities
9 will increase the demand for more luxurious houses.
10
11
12
13

14 These potential structural changes within the regions of cluster 1 are consistent with
15 all of our findings. First, the increase in housing quality will result in a larger increase
16 in the average house prices in class 1 as compared to class 2. Our second finding is that
17 house prices in these regions react faster to changes in GDP. This may be caused by the
18 fact that the increase of their income may influence the decision of these individuals to
19 move and start commuting. Our last and most striking finding is that the house prices
20 in class 1 increase almost twice as fast as GDP. Note however that the increase is not
21 corrected for higher housing quality.
22
23
24
25
26
27
28
29
30
31
32

33 4.3 Impulse-response functions

34 To give further interpretation to our estimation results we compute impulse-response
35 functions for two interesting scenarios, each occurring in the second quarter of 2005. In
36 the first scenario real GDP receives a shock of 1%. In the second scenario real GDP
37 stays the same, but the interest rate receives a shock of 1%-point. We forecast the house
38 prices for each of the scenarios and compare with a no-change scenario, for the subsequent
39 three-year period from 2005Q3 until 2008Q2.
40
41
42
43
44
45
46
47

48 In order to compute the impulse responses up to 12 quarters ahead, we also need
49 forecasts for GDP and the interest rate, as these variables also affect house prices, see
50 (1). Here we assume that the interest rate stays the same during the forecast period. In
51 scenario 3, the interest rate is higher, but still assumed to be constant over the whole
52 forecast period. To obtain forecasts for GDP we construct a simple $AR(q)$ model with
53 intercept for $\Delta \log y_t$. We choose q based on out-of-sample forecasting performance, where
54
55
56
57
58
59
60

1
2
3
4
5
6 we use the last 3 years as a hold-out sample. It turns out that $q = 8$ gives the best
7
8 performance.
9

10
11 [Figure 6 about here.]
12
13

14
15 Figure 6 shows the impulse-response functions of the log house prices with respect
16 to the log of GDP. The y -axis gives the relative change in house prices between the two
17 scenarios, that is, a value of 0.01 means that the house price is 1% higher than the
18 reference forecast. We calculate the impulse response functions for each of the 76 regions.
19
20 We then aggregate these to average responses in the two clusters.
21
22
23

24
25 We find that the effect of an increase in GDP is initially negative in both clusters,
26 which is caused by the many negative $\alpha_{1,i}$ parameter in both clusters. However, this
27 negative effect lasts only one quarter, and after that the house prices are higher compared
28 to the reference forecasts. As expected, we find that the house prices in cluster 1 react
29 both faster and more on the change in GDP.
30
31
32
33
34

35
36 [Figure 7 about here.]
37
38

39
40 In the second scenario, the interest rate receives a shock, and increases from 2.06% to
41 3.06%. We find that the house prices are falling. After three years the house prices are
42 about 2% lower in lower in cluster 2 and almost 3% lower in cluster 1, as compared to
43 the reference forecasts.
44
45
46
47
48

49 50 51 52 53 54 55 56 57 58 59 60

5 Conclusions

In this paper we developed a latent-class panel time series model for describing several key characteristics of regional house prices in the Netherlands between 1985 and 2005. An important feature of the model is that we cluster the regions in separate classes, where the price dynamics of house prices in regions within the same class are similar, while they

1
2
3
4
5
6 are different across the classes. For the 76 regions in the Netherlands we find that two
7
8 classes are sufficient. The first class contains mainly rural regions close to large cities.
9
10 The second class contains both the larger cities and some more remote rural regions.
11
12 The house prices in regions in the first class are characterized by slightly higher average
13
14 growth rates, and stronger and faster reactions to changes in GDP. These findings may
15
16 be caused by the increased number of commuters. Indeed, the number of people working
17
18 in the larger cities, but living in the regions of class 1, has increased substantially during
19
20 our sample period.
21

22
23 Our model allows for the analysis of rather detailed data. To fully exploit its properties
24
25 one would want to analyze even further disaggregated data. The collection of such more
26
27 detailed series is left to further research. Another issue for further research is to make the
28
29 class probabilities dependent on certain explanatory variables.
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60

References

- Andrews, R. L. and I. S. Currim (2003), A Comparison of Segment Retention Criteria for Finite Mixture Logit Models, *Journal of Marketing Research*, **40**, 235–243.
- Boswijk, H. P. (1994), Testing for an Unstable Root in Conditional and Structural Error Correction Models, *Journal of Econometrics*, **63**, 37–60.
- Bozdogan, H. (1987), Model Selection and Akaike's Information Criterion (AIC): The General Theory and its Analytical Extensions, *Psychometrika*, **52**, 345–370.
- Bozdogan, H. (1994), Mixture-Model Cluster Analysis Using Model Selection Criteria and a New Information Measure of Complexity, in H. Bozdogan (ed.), *Proceedings of the First US/Japan Conference on the Frontiers of Statistical Modeling: An Informational Approach*, vol. 2, Kluwer, Boston.
- Breitung, J. and M. H. Pesaran (2008), Unit Roots and Cointegration in Panels, in L. Matyas and P. Sevestre (eds.), *The Econometrics of Panel Data (Third Edition)*, Kluwer, Boston.
- Cameron, A. C. and P. K. Trivedi (2005), *Microeconometrics*, Cambridge University Press, New York.
- Cameron, G., J. Muellbauer, and A. Murphy (2006), Was there a British House Price Bubble? Evidence from a Regional Panel, Economics Series Working Papers 276, University of Oxford, Department of Economics.
- Davies, R. B. (1977), Hypothesis Testing when a Nuisance Parameter is Present Only Under the Alternative, *Biometrika*, **64**, 247–254.
- Dempster, A. P., N. M. Laird, and D. B. Rubin (1977), Maximum Likelihood Estimation from Incomplete Data via the EM Algorithm, *Journal of the Royal Statistical Society, Series B*, **39**, 1–38.
- Doornik, J. A. (2007), *Object-Oriented Matrix Programming using Ox*, 3rd edn., Timberlake Consultants Press, London.
- Franses, P. H. and R. Paap (2001), *Quantitative Models in Marketing Research*, Cambridge University Press, Cambridge.
- Holly, S., M. H. Pesaran, and T. Yamagata (2008), A Spatio-Temporal Model of House Prices in the US, *Journal of Econometrics*, forthcoming, Cambridge Working Papers in Economics No. 654, University of Cambridge, and CESInfo Working Paper Series No. 1826.
- Im, K. S., M. H. Pesaran, and Y. Shin (2003), Testing for Unit Roots in Heterogenous Panels, *Journal of Econometrics*, **115**, 53–74.

- 1
2
3
4
5
6 International Monetary Fund (2004), The Global House Price Boom, *World Economic*
7 *Outlook*, 1491–1517.
8
9
10 Levin, A., C. F. Lin, and C. J. Chu (2002), Unit Root Tests in Panel Data: Asymptotic
11 and Finite-Sample Asymptotics, *Journal of Econometrics*, **108**, 1–24.
12
13 Malpezzi, S. (1999), A Simple Error Correction Model of House Prices, *Journal of Housing*
14 *Economics*, **8**, 27–62.
15
16 Meng, X.-L. and D. B. Rubin (1993), Maximum Likelihood Estimation via the ECM
17 Algorithm: a General Framework, *Biometrika*, **80**, 267–278.
18
19 Moon, H. R. and B. Perron (2004), Testing for a Unit Root in Panels with Dynamic
20 Factors, *Journal of Econometrics*, **122**, 81–126.
21
22
23 Paap, R., P. H. Franses, and D. J. C. Van Dijk (2005), Does Africa Grow Slower Than Asia,
24 Latin America and the Middle East? Evidence from a New Data-Based Classification
25 Method, *Journal of Development Economics*, **77**, 553–570.
26
27
28 Pesaran, M. H. (2004), General Diagnostic Tests for Cross Section Dependence in Pan-
29 els, Cambridge Working Papers in Economics No. 435, University of Cambridge, and
30 CESInfo Working Paper Series No. 1229.
31
32 Pesaran, M. H. (2007), A Simple Panel Unit Root Test in the Presence of Cross-Section
33 Dependence, *Journal of Applied Econometrics*, **22**, 265–312.
34
35
36 Pesaran, M. H., A. Ullah, and T. Yamagata (2008), A Bias-Adjusted LM Test of Error
37 Cross Section Independence, *The Econometrics Journal*, **11**, 105–127.
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60

A Regions by number

1	Noordoost-Groningen	27	Kop v. Noord-Holland	52	Dordrecht
2	Slochteren +s	28	Noord-Kennemerland	53	Gorinchem
3	Grootegast +s	29	West-Friesland	54	Culemborg/Dodewaard
4	Stad Groningen +s	30	Midden-Kennemerland	55	Ede +s
5	Zuidoost-Groningen	31	Waterland	56	Arnhem
6	Noord-Drenthe	32	Zaanstreek	57	Duiven/Westervoort
7	Opsterland	33	Zuid-Kennemerland	58	Elst +s
8	Oost-Friesland	34	Amsterdam	59	Nijmegen
9	Noordwest-Friesland	35	De Bollenstreek	60	Noordoost-Brabant
10	Zuidwest-Friesland	36	Haarlemmermeer	61	Uden +s
11	Zuid-Friesland	37	Almere	62	Oss +s
12	Zuidwest-Drenthe	38	Het Gooi	63	Den Bosch
13	Zuidoost-Drenthe	39	Amersfoort	64	Waalwijk/Drunen
14	Hardenberg +s	40	Barneveld	65	Zeeuwse Eilanden
15	Kop van Overijssel	41	Bunnik/Zeist	66	Zeeuws-Vlaanderen
16	Zwolle +s	42	Utrecht	67	Bergen op Zoom +s
17	Raalte +s	43	Woerden	68	West-Brabant
18	Almelo Tubbergen	44	Alphen	69	Breda
19	Hengelo Enschede	45	Leiden	70	Tilburg/Oirschot
20	Ruurlo Eibergen	46	Den Haag	71	Eindhoven +s
21	Doetinchem +s	47	Gouda	72	Zuidoost-Brabant
22	Zutphen +s	48	Delft +s	73	Noord-Limburg
23	Apeldoorn +s	49	Rotterdam	74	Weert +s
24	Nunspeet +s	50	Westland	75	Roermond +s
25	Lelystad	51	Brielle/Goeree	76	Zuid-Limburg
26	Den Helder/Texel				

Note: +s means including surrounding area.

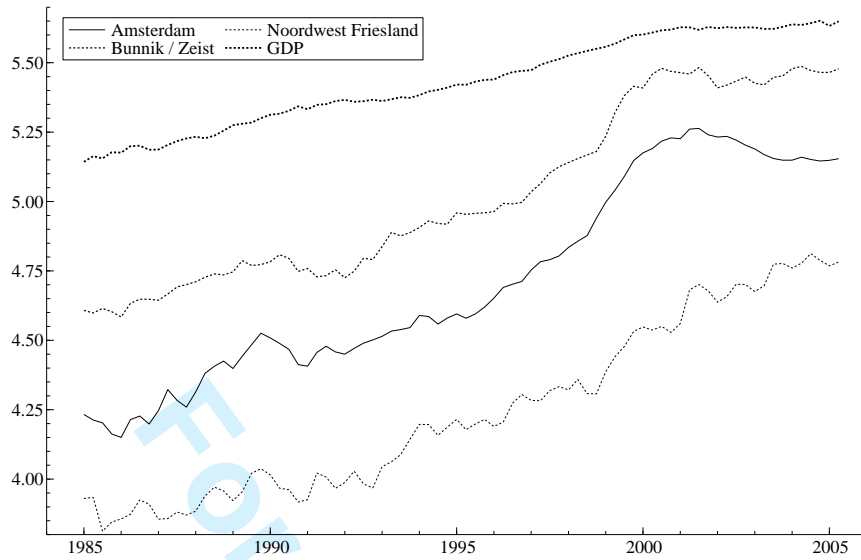


Figure 1: Log house prices for 3 distinct regions, and log GDP.

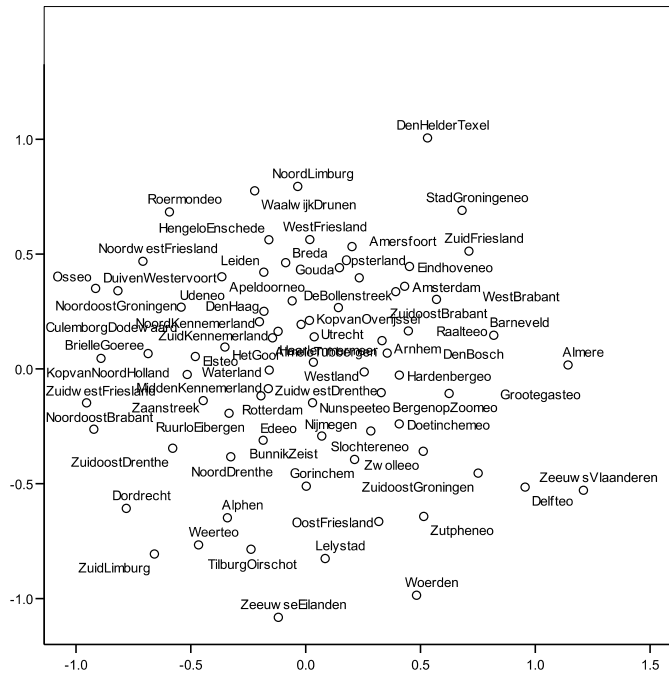


Figure 2: Multidimensional scaling plot of the regions, based on the correlations of the first differences of the log house prices over the period 1985Q1-2005Q4.

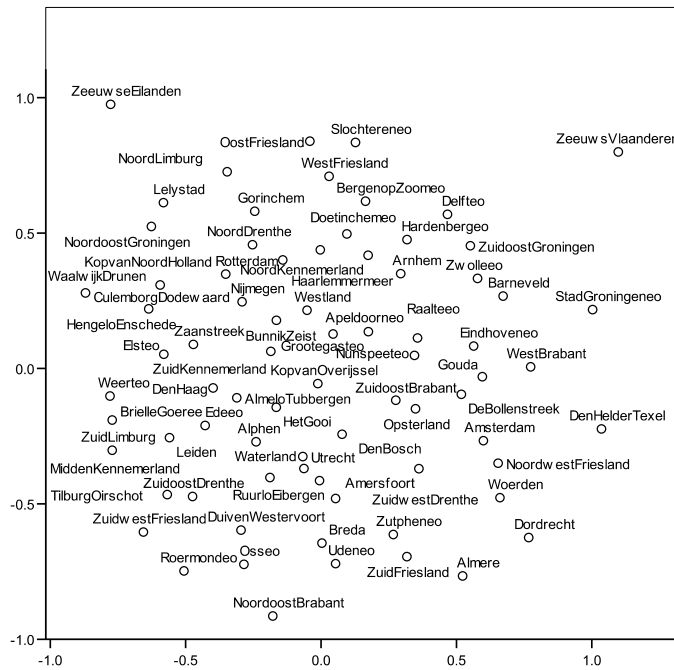


Figure 3: Multidimensional scaling plot of the regions, based on the correlations of the residuals of the ADF regressions for the log house prices over the period 1985Q1-2005Q4.

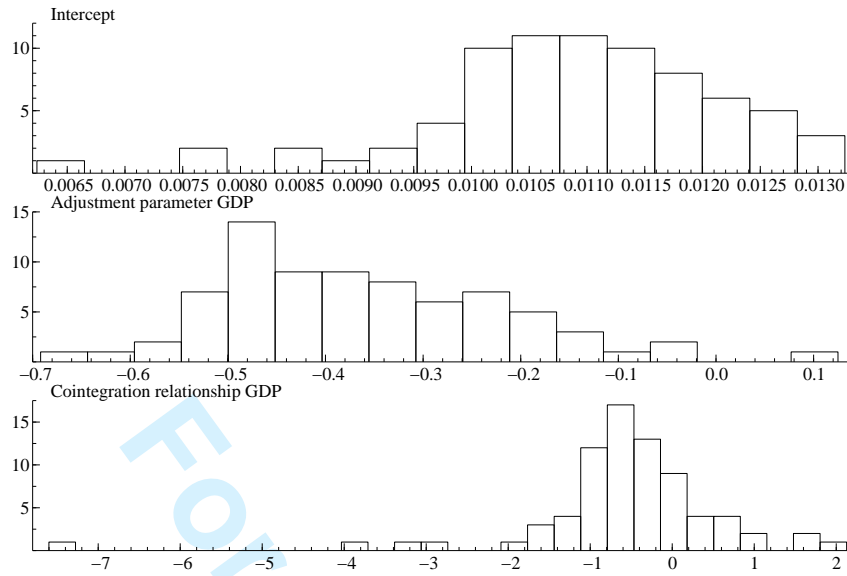


Figure 4: Histograms of the estimated values of the parameters β_j , $j = 0, 1$, and γ in (1) in the fully heterogenous model with 76 classes.

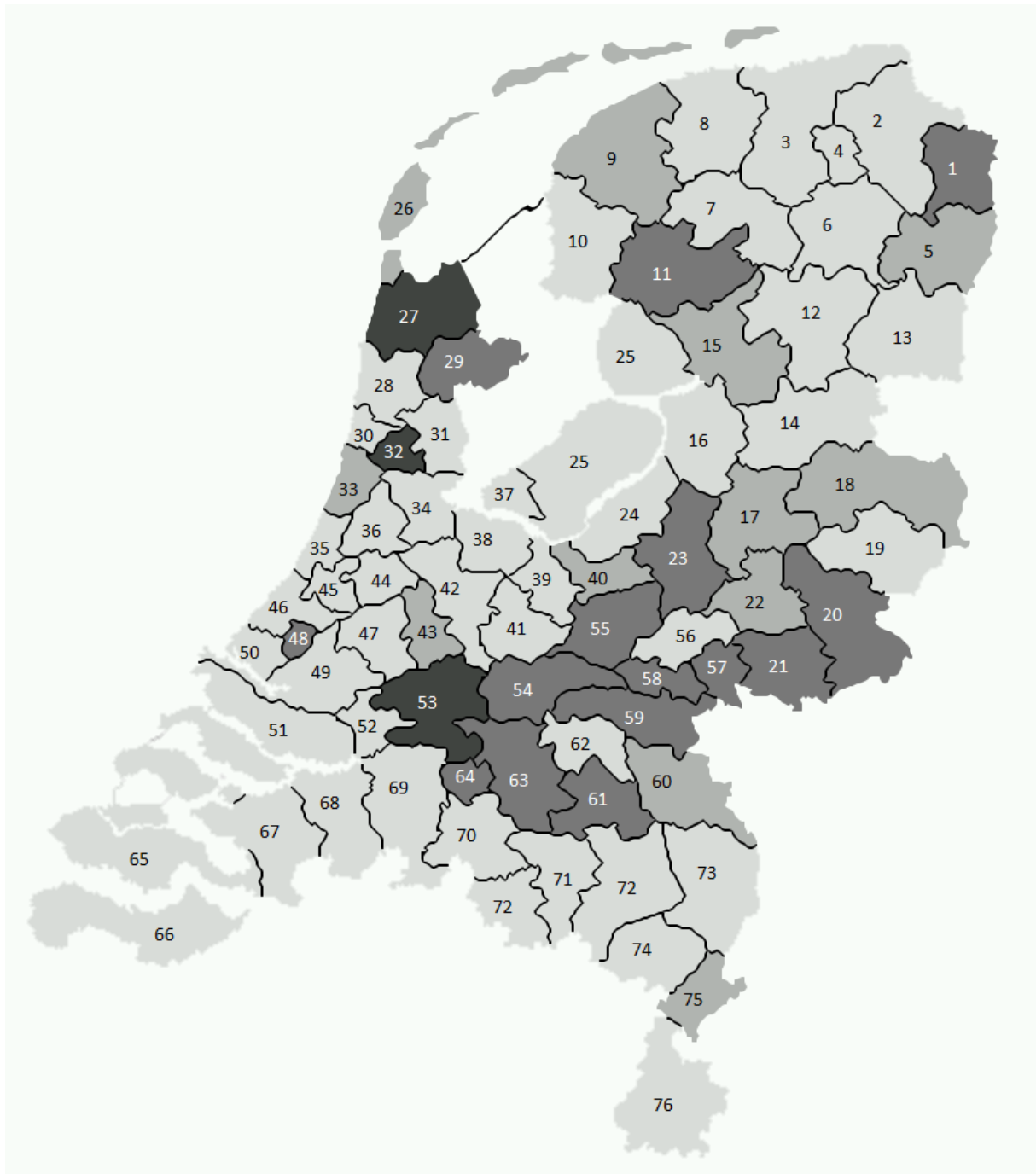


Figure 5: Clustering of regions. Regions with a high probability of belonging to class 1 are colored dark, regions with a low probability of belonging to class 1 are colored lighter. The numbers inside the regions correspond to the ones in Appendix A.

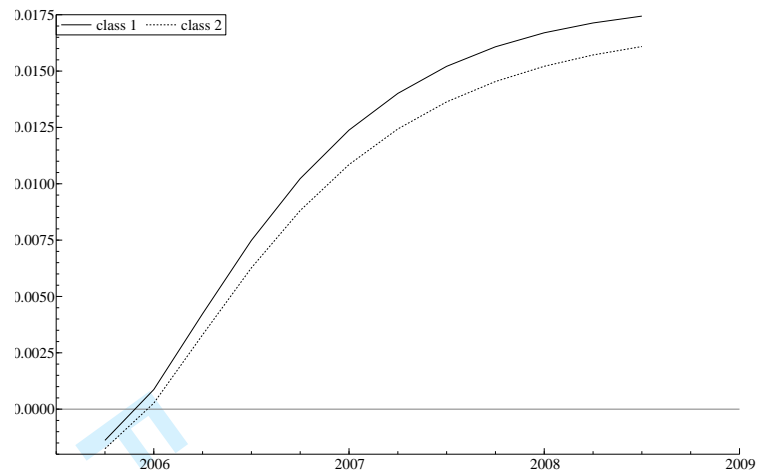


Figure 6: Impulse-response function of $\log(p_{i,t})$ with respect to $\log(y_t)$ for 3 regions.

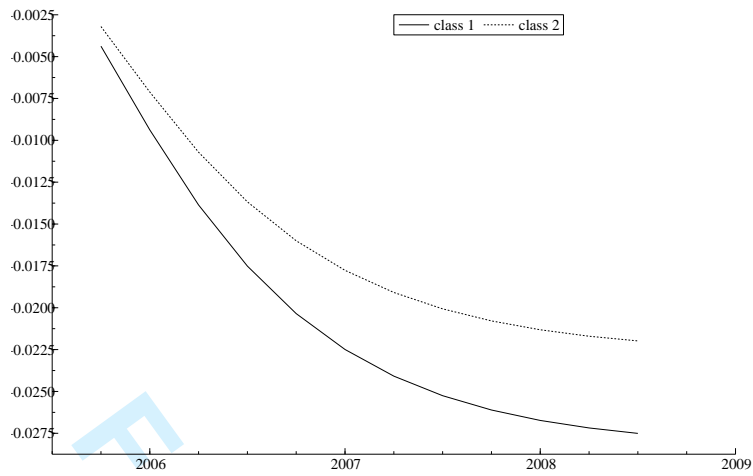


Figure 7: Impulse-response function of $\log(p_{i,t})$ with respect to $\log(I_t)$ for 3 regions.

Table 1: Results of the CD test, the LM_{adj} test and three different tests for a unit root for two series (boldface numbers indicate rejection of the null hypothesis).

Test	Series	$\Delta[\log(p_{i,t})]$	$\log(p_{i,t}) - \log(p_{34,t})$
CD ^a		92.0	144.2
LM_{adj} ^a		60.4	175.1
LLC ^a		-61.2	2.0
IPS ^a		-55.9	1.9
CIPS ^b		-8.9	-3.5

^a Test statistic is asymptotically distributed as normal

^b Tables with critical values for various values for N and T are given by Pesaran (2007), in the presence of an intercept and a trend in the CADF equations and for $N = T = 70$ the critical value at the 95%-level is -2.58 , for $N = T = 100$ it is -2.56 .

Table 2: Results for the fully heterogenous model.

Parameter	Mean	Minimum	Maximum
β_0	0.011	0.006	0.013
β_1	-0.363	-0.692	0.125
γ	-0.591	-7.601	2.127
α_1	-0.271	-1.851	0.703
α_2	-0.004	-0.013	0.004
α_3	0.277	-0.340	0.739

For Peer Review

Table 3: Criteria values for different numbers of latent classes (boldface numbers indicate the optimum).

Criterion \ K	1	2	3	4	76
AIC	-3.937	-4.059	-4.058	-4.057	-3.962
AIC-3	-3.887	-4.008	-4.006	-4.004	-3.862
BIC	-3.598	-3.716	-3.710	-3.704	-3.292
CAIC	-3.584	-3.665	-3.658	-3.652	-3.192

For Peer Review

Table 4: Estimation results for $K = 2$ latent classes.

Class	Estimate	Standard error
<i>intercept $\beta_{0,k}$</i>		
1	0.012	0.001
2	0.011	0.000
<i>adjustment parameter GDP $\beta_{1,k}$</i>		
1	-0.178	0.019
2	-0.131	0.007
<i>cointegration relationship GDP γ_k</i>		
1	-1.888	0.083
2	-1.684	0.035
<i>mixing proportions π_k</i>		
1	0.202	0.159
2	0.798	.

Table 5: Wald tests for equality of the parameters across the two classes in (1).

Restriction	Wald statistic	p-value
$\beta_{0,1} = \beta_{0,2}$	0.61	0.41
$\beta_{1,1} = \beta_{1,2}$	7.10	0.01
$\gamma_1 = \gamma_2$	6.90	0.01

For Peer Review

Table 6: Cointegration relationships between the regions from clusters i and j .

$i \setminus j$	1	2
1	(1, -1)	(1, -1.12)
2	(1, -0.89)	(1, -1)

For Peer Review