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Merit, Approbation and the Evolution of Social Structure*

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Abstract

We examine a society in which individuals gain utility from income and social approbation. Approbation is given to an unobservable trait, signalled through social mobility. Two environments are studied: in one players care for absolute approbation; in the other relative approbation matters. In both environments, individuals' quest for approbation both affects and is affected by social structure. We study the long run implications of that interaction on social organization. Various forms of dynasties and meritocracies are possible. Even though social mobility is driven purely by meritocratic principles, pure dynasties can emerge.

1 Introduction

Most people get value from the approbation of others. The idea that our peers believe we are "special" in some (positive) way tends to increase our utility levels. At the same time there is a fairly natural tendency to grant this approbation when someone does seem special. Different societies, and one society at different points in time, have different ideas about what counts as special in this regard. In a caste society or an old aristocracy, blood

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is the key. Approbation is granted to those born into a high caste, and contempt to those born in a low caste. In a pure meritocracy by contrast, approbation is granted to those with merit: those who have skills and/or put in high effort.[1] Most societies are some mix of both notions, with the general thought that approbation based on merit is somehow more modern.

Some sources of approbation are readily visible: skill at basketball or movie acting, for example, or the social status of one's forebears. Others, such as intelligence, are not. The former can simply be displayed; the latter must be signalled by some other means. One means of signalling high skills of various kinds is to enter a social or income class that differs from one's origin class, for example through a particular profession. Social approbation follows. This is what this paper is about. Individuals try to signal their talent through the application of effort, which can result in a class change and the acquisition (or loss) of approbation. As approbation tends to be limited (if some people get more then some others should get less) the quest for approbation has an impact on social structure, while at the same time social structure might influence the conditions of approbation acquisition. The analysis of that interaction is the object of this paper.

1.1 Approbation

The observation that individuals seek approbation from their peers is not new. In the literature on conspicuous consumption, for instance, public approbation is granted for the ownership of large wealth. Because the latter is not directly observable it must be signalled by the consumption of a positional good.[2] In that literature, what matters is relative rather than absolute position. In Frank (1985b), Fershtman et al. (1996), Hopkins and Kornienko (2004) or Robson (1992), an agent's position in the population distribution over some variable enters the utility function directly. Frank (1985b) assumes an individual's utility increases with his quantile position in the distribution of consumption levels of the positional good. Robson, to examine risk taking behaviour, assumes that utility is determined by an agent's wealth and his ordinal rank in the wealth distribution. Hopkins and Kornienko use a similar positional approach to study the effects of income distribution on excess "conspicuous consumption" and derive the effect of exogenous changes in the income distribution. Fershtman et al. study the effects of income distribution on talent allocation and growth when an individual cares about the average level of human capital in his occupation relative to that in the alternative occupation.

An alternative approach does not "put the others" directly in the utility function, but lets the concern for status emerge from some non-market tournament whose outcome enters the utility function (such as access to social circles, clubs, marriage, and so on). In Cole et al. (1992) and Corneo and Jeanne (1998, 1999), social recognition obtains from marrying the right (high quality) mate or getting one's offspring to marry the right mate, the probability of which is determined by relative position again. In this tradition, desire

[1] "Meritocracy" was coined by Young (1958). He defined it as a system in which recruitment to social positions is based solely on merit, defined as the combination of ability and effort.

[2] For early insights into conspicuous consumption see Marshall (1890) and Veblen (1899). A comprehensive survey on status and economic decisions is Weiss and Fershtman (1998). For stimulating discussions on the competition for approbation, status and other scarce social resources see Hirsch (1976) and Frank (1985a).

for status is instrumental, arising endogenously from agents' attempts to manipulate a different argument of the utility function.[3]

Much of this literature concludes that in general the competition for status positions is self-stultifying: if all agents make equivalent attempts to move up the hierarchy (buying cars that are twice as big as their current ones), the result is simply an increase in the resources spent in signalling and no relative movement of agents within the hierarchy. Thus even if this behaviour generates growth in output, it generally does not raise aggregate welfare and tends to constitute a social waste. Ireland (1998), for example, presents this argument, and argues that a tax can be an effective response to this over-consumption problem.[4]

1.2 Mobility

In the model we develop below, mobility is treated as intrinsically meritocratic: if an agent has ability and works, he or she will rise (or stay in the top). Thus in terms of motivation (incentives) agents live in a meritocratic system in the sense that social promotion will take place if an agent applies effort to skill, and demotion will happen if effort and skill are lacking. This is the type of regime observed in the empirical sociology literature. In recent years the discussion there has moved from whether or not a society *is* meritocratic to whether the degree to which it is a meritocracy has changed (see the introduction to Breen and Goldthorpe 2001). On one side, Saunders (1997) and Bond and Saunders (1999), for example, contend that now in the U.K. a person's eventual social status is determined by his or her intelligence and effort (following Young's initial definition quite closely). On the other side, Savage and Egerton (1997) show that high ability, high status children are likely to be high status adults, whereas low ability, low status children are still likely to be low status adults: specifically, the probability of ending in the service (upper) class is equal for low ability/high status children and high ability/low status children (about 30 percent). Similarly, Breen and Goldthorpe (1999) find that origin class continues to play a strong role. The difference falls when we correct for merit, and merit variables are statistically significant, but they conclude that children from lower classes need to show considerably more merit to enter a particular (higher) class than do higher class children (see also Boudon 1974). The empirical literature has had a focus on the correlation between the status (or income) of a child and that of its parent, but typically correcting for the effects of meritocratic variables is statistically significant.[5] Even those authors who contend that data do not show meritocracies in the strongest sense of Young find that ability and

[3]Another interesting approach is provided by Ireland (1998), who introduces "others" into preferences by making an agent's utility a convex combination of his or her own fundamental utility and spectators' estimate of that utility.

[4]We must also note Corneo and Jeanne (1998) who find that status-seeking can be beneficial, provided it happens early in the life cycle. It is also possible to find a link between income distribution and status-seeking activities, whereby differences in taste for status among rich and poor induce different investment decisions (regarding for instance schooling) which can in turn further affect income distribution and growth. See the survey by Weiss and Fershtman (section 3.4) on these issues.

[5]In addition to the authors cited, see for example Deardon et. al (1997) and Johnson and Reed (1996). In general, one of the problems that arises (and is admitted by participants) in this debate is the difficulty of defining and measuring "ability" and "effort". For more on how these forces affect social mobility more generally, see for example Treiman and Ganzeboom (1990), Breen and Goldthorpe (2001), Deardon et al. (1997) and Erikson and Goldthorpe (2002).

effort do have effects on destination class.

In the current paper we build on a framework initially presented in Piketty (1998). In that paper social approbation is given to an unobservable trait which must be signalled through an agent's social mobility. Social mobility thus contributes to utility in two ways: directly, through its effect on income, and instrumentally, in its contribution to public approbation. This gives agents an incentive to try to enter (or to stay in) the high income class. Mobility is driven by a simple mechanism involving origin, effort and ability, as discussed in the empirical literature on social mobility just referred to. Based on observed social mobility, society forms beliefs about who deserves approbation, thereby creating a social contest. Individual effort-provision decisions thus have collective implications on class composition.[6]

Our model simplifies Piketty's general structure by making effort levels discrete and considering a more compact mobility "production function". We add a dynamic structure in which agents are born into a social class, but each generation makes its own effort decisions. This creates a repeated one-shot game in a changing environment. Depending on the importance of the approbation motive and the prevalence of talent, we show that a variety of organizations are supported, involving both dynastic and meritocratic elements. One interesting result is that under identifiable conditions a purely dynastic structure can still emerge, in which all agents remain in the social class of their parents. Here, not only is there zero structural mobility, but the much stronger condition that there is zero exchange mobility is also present.[7] This suggests that defining or identifying a meritocracy may not simply be a matter of observing inter-generational (im)mobility. In the debate between Saunders (1997) and Breen and Goldthorpe (1999), for example, much of the argument is about defining the appropriate measure for inter-class mobility.[8] Whether or not we observe meritocratic mobility may well turn on these definitions, but our model demonstrates that a society in which all promotion is done explicitly on meritocratic principles can still have a frozen social structure with no inter-generational mobility at all. Given that, the debate about whether a society is or is not meritocratic cannot necessarily be solved by agreeing on the definition of mobility.

In the second part of the paper we introduce the notion of relative approbation, referring to the idea that agents care not about absolute approbation levels but rather about their standings in the approbation distribution. That competition for relative approbation (in the manner of Frank and the papers discussed above) creates a global feedback from the class structure to the value of approbation. While competition for absolute approbation

[6]Piketty's concern revolves around the possibility that in two equivalent societies there can be one in which agents actively seek status whereas in the other status-seeking behaviour is absent. Besides multiplicity, Piketty also discusses the way the quest for status can amplify initial inequalities.

[7]"Structural mobility" refers to net mobility and the change in relative sizes of classes; "exchange mobility" refers to gross mobility, and whether some agents change class (possibly offset by other agents changing in the opposite direction)..

[8]Saunders argues that absolute, rather than relative mobility is a preferred measure, but if relative mobility is under discussion then disparity ratios (which measure the chances that individuals of different origin classes are found in a particular destination class) rather than odds ratios (which measure those chances in comparison with those same individuals being found in some other class) should be used. Saunders argues that using absolute mobility or disparity ratios indicate that Britain is meritocratic to a great extent, whereas Breen and Goldthorpe argue that odds ratios show that it is not.

was essentially intra-class, competition for relative approbation becomes inter-class. In this case too much upward mobility reduces the value of the approbation gained by high class individuals. While not reducing the income associated with the upper class, it reduces the net inducement to attempt to enter, and thus changes the amount of entry to it. In general, this provides a regulating effect on mobility and class structure. Class structure and effort thereby become jointly endogenous. The effect of the approbation motive on long run social organization in this context is not the same as what we observed in the absolute approbation environment, and the welfare conclusions also differ.

2 The model

Society consists of two locations, $\{0, 1\}$, which correspond to the lower and upper classes respectively. It is populated by a continuum S of risk-neutral individuals having total measure normalized to unity. Let x designate the measure of individuals who belong to the upper class. Each individual $i \in S$ lives for one period during which he has a chance to move between the two classes. Social mobility for i is influenced by three factors: his origin $\ell_i \in \{0, 1\}$, his ability $\alpha_i \in \{a, A\}$ (high or low) and the level of effort, $e_i \in \{e, E\}$ (also high or low).

Individuals' origins are inherited without error from their parents. Abilities are identically and independently distributed: independently from the abilities of other agents, and independently of the social class of the agent.[9] Specifically α_i is high with probability $p \in (0, 1)$, which is known to everyone. Thus there is inter-generational inertia in social class, but none in ability. Finally effort, alone, is under the control of the agent. Depending on these three factors an individual can stay in his origin class, be promoted to the higher class or be demoted to the lower class.

Within a single time period, and knowing his origin class and own ability, an agent chooses an effort level to maximize expected utility. He then moves from his origin to destination class (which may be the same) and collects the income of that class. Finally the agent dies after having produced, into the same class, a single offspring. This process then iterates.

2.1 Utility

Utility is driven by net income and public opinion about ability. The former is determined by an agent's destination class and effort level; the latter is inferred from publicly observable social mobility, itself determined by origin, ability and effort. Utility for any agent i is written

$$r(z_i) - c(e_i) + \lambda \Pr\{\alpha_i = A|\cdot\},$$

with λ the importance of approbation, $c(e_i)$ the cost of producing effort e_i with the normalization that $c(E) = c$ and $c(e) = 0$, and $r(z_i)$ the income, again normalized so that $r(0) = 0$ and $r(1) = 1$. Finally $\Pr\{\alpha_i = A|\cdot\}$ is the posterior probability placed by society on i being of high ability conditional on the publicly observable information. The appro-

[9] In Section 3.3 this assumption is discussed in greater detail.

bation collected by an agent is equal to the posterior probability of that agent being high ability and is determined by Bayes' rule. This is made more precise below.

2.2 Mobility

There are three traits — origin, ability, effort — and each exerts a positive influence on the probability that an agent ends up in the upper class. We adopt a (stochastic) majority rule for social mobility. If an agent moves, it is to the class corresponding to the majority of his three attributes: origin, ability and effort. Thus the possibility of class change only arises for high-effort, high-ability lower class (who can move up) and low-effort, low-ability rich (who can fall). Mobility is not deterministic though: a high-effort, high-ability poor agent rises with probability u, while a low-effort, low-ability rich agent falls with probability d, as summarized in Table 1.

Origin ℓ_i	Ability α_i	Effort e_i	Social mobility $\Pr\{z_i = 1 \| \ell_i, \alpha_i, e_i\}$
1	A	E	1
1	A	e	1
1	a	E	1
0	A	E	u
1	a	e	$1-d$
0	A	e	0
0	a	E	0
0	a	e	0

Table 1: The probabilistic mechanism driving social mobility.

The parameters u and d determine up and down mobility in a transparent way. However, if $u = d = 1$ the mobility rule becomes deterministic, and mobility for the lower class or immobility for the upper class would fully reveal abilities. To avoid this unrealistic (and uninteresting) case, we assume that $u, d \in (0, 1)$.

Which social organizations does the mobility mechanism in Table 1 permit? Because we have included origin class as a determinant of mobility, a *pure meritocracy* (as defined by Young, see Footnote 1), in which social position is determined solely by merit, is not possible.[10] In our structure inertia is introduced through origin class and has the effect of making effort and talent substitutes for the upper class but complements for the lower.

[10]That is, in a pure meritocracy the upper class would contain only and all people who are both high-skilled and who put in high-effort; the lower class would contain all others. In a more general setting, the highest class would contain those with the highest skills and highest effort; the lowest class would contain those with the lowest skills and lowest effort, and intermediate classes would contain intermediate levels of skill and effort, in appropriate combinations. As we have only two social classes, our structure is somewhat less nuanced.

Thus it will always be possible for the talented rich to be idle and still maintain his rank, while the talented idle poor will never leave the lower class. Thus outcomes will be at best partial meritocracies, which we define here. In the terminology we use below, a *positive meritocracy* rewards high-skilled, high-effort agents with upward mobility. A *negative meritocracy* punishes low-effort, low-skilled agents with downward mobility. We also will use *meritocracy* to refer to a mixed situation in which high-skilled, high-effort agents rise while low-skilled, low-effort agents fall. Finally, a *dynasty* is a social structure in which no agent leaves the social class in which he was born. These terms all refer to "observable" social-mobility structures and make no reference to the underlying principles that determine mobility. One question is whether rational responses to individual incentives will permit different regimes to emerge.

3 Equilibrium strategies and stationary structures

Inferences about ability and thus optimal behaviour are affected by what is public and what is private knowledge. We focus on the case in which an individual knows both his ability and his effort, but neither is publicly observable. By contrast social mobility, which serves as a signal of ability, is publicly observable. Alternative information regimes are briefly discussed at the end of the section.

3.1 Equilibrium strategies

The environment considered here has a continuum of players who can be one of 4 types, defined by origin and ability, and play one of two possible pure strategies.[11] Let the 4-tuple $q = (q_{0,A}, q_{0,a}, q_{1,A}, q_{1,a})$ be the proportion of agents of each type who expend high effort. For given q and under the assumption of unobservable effort and ability, application of Bayes' rule gives the posterior probability for a low origin agent i to be high-ability as

$$\begin{aligned}\Pr\{\alpha_i = A|0,0,q\} &= \frac{p - puq_{0,A}}{1 - puq_{0,A}},\\ \Pr\{\alpha_i = A|0,1,q\} &= \begin{cases} 1 & \text{if } q_{0,A} > 0, \\ 0 & \text{if } q_{0,A} = 0. \end{cases}\end{aligned} \tag{1}$$

Similarly the posterior probability for a high origin agent i to be high-ability is

$$\begin{aligned}\Pr\{\alpha_i = A|1,1,q\} &= \frac{p}{1 - d(1-p)(1-q_{1,a})},\\ \Pr\{\alpha_i = A|1,0,q\} &= 0.\end{aligned} \tag{2}$$

Note that if an agent changes class, the signal is definitive, whereas not changing class is only imperfectly informative.

Utility for a high-ability lower class agent is

$$u_{0,A}(e_i, q) = \begin{cases} u\,(1 + \lambda \Pr\{\alpha_i = A|0,1,q\}) + (1-u)\,\lambda \Pr\{\alpha_i = A|0,0;q\} - c & \text{if } e_i = E, \\ \lambda \Pr\{\alpha_i = A|0,0;q\} & \text{if } e_i = e. \end{cases}$$

[11] For general results on games of that class, see for instance Schmeidler (1973).

The same calculations based on the elements in Table 1 and the above posteriors yield similar expressions for the utility levels of the 3 others types of agents. A player's utility depends only on q and the player's own strategy (which does not affect q). Now define $\Delta_{\ell,\alpha}(q) = u_{\ell,\alpha}(E, q) - u_{\ell,\alpha}(e, q)$ as the incentives towards high effort of an agent of origin z and ability α. They are written

$$\begin{aligned} \Delta_{0,a}(q) &= -c, \\ \Delta_{1,A}(q) &= -c, \\ \Delta_{0,A}(q) &= u + \lambda \frac{u(1-p)}{1 - puq_{0,A}} - c, \\ \Delta_{1,a}(q) &= d + \lambda \frac{dp}{1 - d(1-p)(1 - q_{1,a})} - c. \end{aligned} \tag{3}$$

An equilibrium can now be defined. A 4-tuple $q^* = (q^*_{0,A}, q^*_{0,a}, q^*_{1,A}, q^*_{1,a})$ is a pure strategy Nash equilibrium if and only if one of the three following conditions holds true: (i) $q^*_{\ell,\alpha} = 0$ and $\Delta_{\ell,\alpha}(q^*) \leq 0$; (ii) $q^*_{\ell,\alpha} = 1$ and $\Delta_{\ell,\alpha}(q^*) \geq 0$; (iii) $q^*_{\ell,\alpha} \in (0, 1)$ and $\Delta_{\ell,\alpha}(q^*) = 0$, for all $\ell \in \{0, 1\}$ and $\alpha \in \{a, A\}$. Throughout we refer to an equilibrium by the 4-tuple q^* of the proportions of each type that expend E in that equilibrium.

From the equations in (3), low-origin low-ability and high-origin high-ability agents never have incentives to provide E. Thus in an equilibrium $q^*_{0,a} = q^*_{1,A} = 0$. On the other hand both high-origin low-ability and low-origin high-ability agents can prefer high effort over some parameter range(s).[12] Turning to low-origin high-ability agents and setting $\Delta_{0,A}(q) = 0$ yields

$$q_0 = \frac{-\lambda u(1-p) + c - u}{up(c-u)},$$

the asymmetric equilibrium (provided $q_0 \in (0, 1)$). In addition, setting $q_{0,A} = 0$ and $\Delta_{0,A}(q) \leq 0$ gives

$$\lambda \leq \frac{c-u}{u(1-p)} = \lambda_0, \tag{4}$$

while setting $q_{0,A} = 1$ and $\Delta_{0,A}(q) \geq 0$ gives

$$\lambda \geq \frac{c-u}{u(1-p)}(1 - pu) = \Lambda_0. \tag{5}$$

As $\Lambda_0 < \lambda_0$ there is a region of multiple equilibria when $\Lambda_0 \leq \lambda \leq \lambda_0$, where $q^*_{0,A} = 0$, q_0 or 1 are all possible.[13] Similar reasoning for low ability upper class gives

$$q_1 = 1 + \frac{\lambda p}{(c-d)(1-p)} - \frac{1}{d(1-p)}$$

and the critical values of the approbation motive

$$\lambda_1 = \frac{c-d}{pd}(1 - d(1-p)), \tag{6}$$

$$\Lambda_1 = \frac{c-d}{pd}. \tag{7}$$

[12] To ensure that income effects alone are not enough to induce high effort, we assume that $c > \max\{u, d\}$.

[13] The 3 equilibria that exist when $\Lambda_0 \leq \lambda \leq \lambda_0$ are Pareto ranked, $q^*_{0,A} = 0$ yields the largest utility, before q_0 and 1.

This time however $\lambda_1 < \Lambda_1$, thus when $\lambda_1 \le \lambda \le \Lambda_1$ only the asymmetric equilibrium is possible. Equilibrium behaviour in relation to the approbation motive λ is summarized in Proposition 1.

Proposition 1 *Equilibrium is characterized by*

$$\begin{aligned} q^*_{0,a} &= 0, \\ q^*_{1,A} &= 0, \\ q^*_{0,A} &= \begin{cases} 0 & \text{if } \lambda < \Lambda_0, \\ \{0, q_0, 1\} & \text{if } \Lambda_0 \le \lambda \le \lambda_0, \\ 1 & \text{if } \lambda_0 < \lambda, \end{cases} \\ q^*_{1,a} &= \begin{cases} 0 & \text{if } \lambda < \lambda_1, \\ q_1 & \text{if } \lambda_1 \le \lambda \le \Lambda_1, \\ 1 & \text{if } \Lambda_1 < \lambda. \end{cases} \end{aligned}$$

Proof. Direct from the expressions in (3) and the discussion above. ■

Multiplicity for talented lower class arises because in the posterior, mobility is fully informative whereas the informativeness of immobility increases with $q_{0,A}$, yielding incentives towards E that are largest when $q_{0,A} = 1$. This is so because effort and talent are complements in the mobility function (an effect also at work in Piketty). Thus when $\Lambda_0 \le \lambda \le \lambda_0$, the opinion that lack of promotion expresses lack of talent is equally possible as the opinion that lack of promotion merely reflects bad luck. High society agents have a class advantage: for them talent and effort enter the mobility technology as perfect substitutes, doing away with multiple equilibria.

The approbation motive increases the incentives towards E, shifting left the critical λ-values. Other effects of the parameters of the model are easily seen. Increasing the stochastic components of social mobility u and d also shifts left the critical λ-values. The effect of p is class-specific: a talented lower class who is rare (low p) gains a lot by signalling his talent, but when p is large the gain is smaller as everyone is already “presumed” to be smart. By contrast, a talentless high society in a world of widespread talent (large p) who fails to remain high society loses a lot: he is easily singled out as low-ability. Thus a stronger approbation motive is needed to induce high effort from the poor if high-ability is common, whereas for the rich it is when high-ability is rare.

3.2 Evolution of social structure

Observation of modern societies shows not only that exchange mobility, in which individuals move from one class to another, is important, but also that structural mobility, wherein the overall structure of classes changes, can be observed.[14] We study that issue in the overlapping structure that has been set forth, with social class being inherited without error. The rules of mobility in Table 1 have been discussed in detail, but we should note again here that the principle is meritocratic in the following sense: an agent can only rise if he or she has both high skill and high effort; an agent can only fall if he or she has both low

[14] See Erikson and Goldthorpe (1992) for example.

skill and low effort. This principle remains fixed throughout. The conditions under which this meritocratic principle results in any version of a meritocratic observable is examined now.

Social structure will evolve as a discrete time process with effort levels determining social movements. The resulting structure forms the environment in which the next generation makes decisions about effort provision. Let x^t be the measure of the upper class at time t. The evolution of social structure is obtained by considering the rates of up and down mobility. The rate of inflow to high society is $r^+(x) = (1/x - 1)puq^*_{0,A}$, whereas the rate of outflow from high society is $r^-(x) = (1-p)d(1-q^*_{1,a})$. As a result the rate of change in x is simply $r^+(x) - r^-(x)$, and the time evolution of high society is written $x^{t+1} = x^t\{1 + r^+(x^t) - r^-(x^t)\} = T(x^t)$.

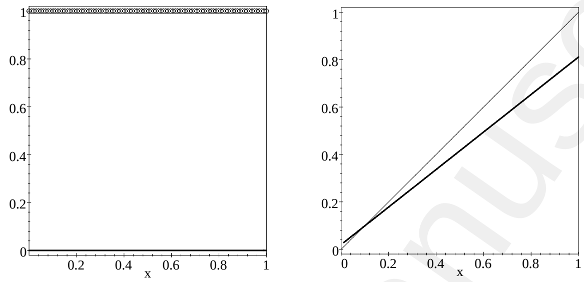

Figure 1: The equilibrium $q^*_{0,A}$ and $q^*_{1,a}$ (left panel) and the map T for $p = .1$, $u = .2$, $d = .21$, $c = .3$ and $\lambda = .6$ (right panel).

In Figure 1 above an illustrative example is provided. Low society always provides E (circles on the left panel) while high society always provides e (thick black curve on the left panel). These parameters yield $x^\infty = .096$ as the unique stationary outcome, which is a *meritocracy.* The proposition below summarizes the results.

Proposition 2 *Consider three cases, as defined by critical values for the upper class.*

- *Suppose* $\lambda < \lambda_1$ *(i.e. low effort is optimal for low-ability high-origin agents). Then*

$$x^\infty = \begin{cases} 0 & \text{if } \lambda < \Lambda_0, \\ \left\{0, \frac{up}{up+d(1-p)}, \frac{u(\lambda p-\lambda-1)+c}{u(\lambda p-\lambda-1)+c+d((c-u)(1-p)}\right\} & \text{if } \Lambda_0 \leq \lambda \leq \lambda_0, \\ \frac{up}{up+d(1-p)} & \text{if } \lambda_0 < \lambda. \end{cases}$$

- *Suppose* $\Lambda_1 < \lambda$ *(i.e. high effort is optimal for low-ability high-origin agents). Then*

$$x^\infty = \begin{cases} x^0 & \text{if } \lambda < \Lambda_0, \\ \{x^0, 1\} & \text{if } \Lambda_0 \leq \lambda \leq \lambda_0, \\ 1 & \text{if } \lambda_0 < \lambda.. \end{cases}$$

- *Suppose* $\lambda_1 \leq \lambda \leq \Lambda_1$ *(i.e. a share* q_1 *of low-ability high-origin agents provides high effort). Then*

$$x^\infty = \begin{cases} 0 & \text{if } \lambda < \Lambda_0, \\ \left\{0, \frac{up}{up+d(1-p)(1-q_1)}, \frac{(u(-\lambda(1-p)-1)+c)(c-d)}{(2u+\lambda-2c)(d-c)+\lambda p(u-d)}\right\} & \text{if } \Lambda_0 \leq \lambda \leq \lambda_0, \\ \frac{up}{up+d(1-p)(1-q_1)} & \text{if } \lambda_0 < \lambda. \end{cases}$$

Proof. When a proportion $1 - q^*_{1,a}$ of low ability high origin agents gives e, they exit from the upper class, causing it to decline at a rate $(1 - p)(1 - q^*_{1,a})d$. If the high-ability low origin agents give high effort in proportion $q^*_{0,A}$, they enter the upper class at a rate $(1/x - 1)pq^*_{0,A}u$. If only the first motion exists (when $\lambda < \lambda_1$ and $\lambda < \Lambda_0$), then the only steady state of the system is $x^\infty = 0$. When only the second motion exists, each period the upper class grows, and $x^\infty = 1$ is the only steady state of the system. In the intermediate cases where both motions exists, the steady state is found by solving $r^+(x) = r^-(x)$, and the proposition follows simply, case by case. ■

Effort levels and the resulting social structure depend on $\lambda_0, \Lambda_0, \lambda_1$ and Λ_1, which, under different parameter values, occupy different positions along the approbation motive axis. There are essentially two general cases: when the lower class thresholds are below the upper class ones, and vice versa. Which situation obtains depends on the prevalence of high ability. As $\partial\lambda_0/\partial p, \partial\Lambda_0/\partial p > 0$ and $\partial\lambda_1/\partial p, \partial\Lambda_1/\partial p < 0$, the first situation $\Lambda_0 < \lambda_0 < \lambda_1 < \Lambda_1$ obtains when talent is rare (low p) whereas the second situation $\lambda_1 < \Lambda_1 < \Lambda_0 < \lambda_0$ is associated with talent being common (high p). Figure 2 summarizes the results for these two polar orderings.[15] The solid segments represent flows out of the lower class into the upper one, while the dashed segments represent flows from the upper class to the lower one.

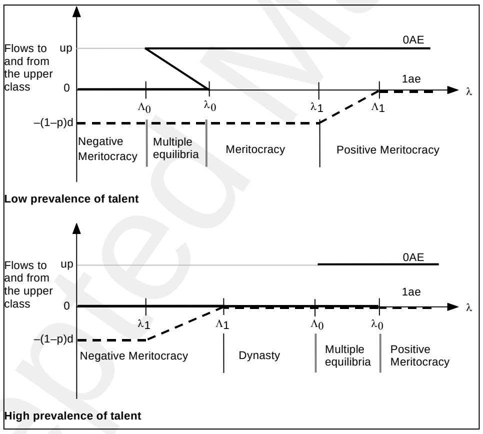

Figure 2: Gross flows into and out of the upper class as functions of the approbation motive λ.

Consider first the extreme cases of very weak or very strong approbation motives. When λ is low all agents give low effort, and the less able members of the upper class (of which each generation has a share $1 - p$) continually move down while no one even attempts to go up. Society is decaying, the mobility function punishes the idle rich who have low ability, the observed social pattern is a *negative meritocracy*. Eventually, by the attrition of those with low ability, and the upper class is reduced to zero, and while agents continue

[15]The three other possible orderings of the roots along the approbation axis yield situations that are similar to but slightly more intricate than the two we present.

to value approbation, the concern for status is no longer manifest or observable since there is no longer any social mobility. By contrast, when λ is high all potentially mobile agents give high effort, and while none leave the upper class, it is continuously entered by lower class high-ability strivers (of which each generation has a share p). The mobility function rewards low-origin, high-ability agents who work hard, the observed social pattern is a *positive meritocracy*. Eventually, though, everyone becomes a member of the upper class, and again approbation-seeking ceases to be visible.

For intermediate λ-values, when talent is rare (upper part in Figure 2) both up and down mobility take place. Though high society converges to a stationary size and structural mobility ceases, systematic turmoil in the social structure, that is exchange mobility, persists. It is a *meritocracy* in that high-ability lower-class workers rise, and low-ability upper-class shirkers fall ($\Lambda_0 < \lambda < \lambda_1$). However, for this regime to be established, high ability must be uncommon. Only when approbation is granted for rare traits will a meritocratic society be observed in practice.

On the other hand, again in the middle ground, when high ability is common (lower part in Figure 2) a *dynasty* appears, in which both upper and lower classes are frozen ($\Lambda_1 < \lambda < \Lambda_0$), and neither structural nor exchange mobility are present. In this case a high-ability, hard-working poor agent could rise, but it is not optimal to expend the effort to do so; a low-ability high-class agent would fall if he were lazy, but he works to prevent it. Thus when approbation is granted for common traits, even though promotion and demotion mechanisms are inherently meritocratic in design, society does not look like a meritocracy, but rather like an archetypal dynasty.

3.3 Discussion

Do the informational constraints bearing on the agents significantly affect the behaviour of the system? Some types of abilities may be difficult even for the agent to observe, and some types of effort may be more obvious than others. Thus in principle there are four possible information settings: the observability of effort crossed with the observability of ability. By re-doing the above calculations under different information regimes, it can be observed that as knowledge asymmetries decrease, the strength of the approbation motive needed to induce high effort increases. It becomes more difficult to induce a change of public perception of ability through social mobility. In the extreme case, where public and private knowledge is identical, that is, where effort is known and ability unknown, no finite utility from approbation is high enough to induce any agent to high effort. Which of the two intermediate knowledge regimes[16] is more conducive to high effort depends on the other parameters.

Symmetric upward and downward mobility ($u = d$) implies that dynasties exist only when high ability is common ($p > 1/2$), while meritocracy exists only when high ability is uncommon ($p < 1/2$). Asymmetric mobility can be examined by fixing u and denoting $p^* = \arg\{\Lambda_1 = \lambda_0\}$. This is the value of p at which an aristocratic regime can appear (recall that meritocracy demands $\Lambda_1 > \lambda > \Lambda_0$, as can be seen in Figure 2). Differentiation

[16]The two intermediate regimes are public and private ignorance of ability with public ignorance but private knowledge of effort; and public and private knowledge about effort with public ignorance and private knowledge about ability.

yields $\partial p^*/\partial d < 0$, that is, increasing the relative ease of downwards mobility *increases* the range of p-values compatible with an aristocracy. Now denote $p^* = \arg\{\Lambda_0 = \lambda_1\}$, the value of p at which a meritocracy can emerge. Again $\partial p^*/\partial d < 0$; that is, increasing the relative ease of downwards mobility *decreases* the range of p-values compatible with a meritocracy. Thus as downward mobility becomes stronger, observing a dynastic structure becomes more likely. As one would expect, the size of the upper class decreases with the relative ease of downward mobility ($\partial x^\infty/\partial d < 0$). The effect of u is the opposite. (In addition, almost everywhere $\partial x^\infty/\partial p > 0$, that is, the size of the upper class increases with the prevalence of high ability) Also note that $u > d$ implies that $x^\infty > p$, while if $d > u$ then $x^\infty < p$. Put simply, if upward mobility is (relatively) easy, then the high class will accommodate all of the high-ability agents and some of the low-ability well-born. If upward mobility is (relatively) difficult, then high-ability low-origin agents cannot rise, and the upper class does not contain all of the high-ability agents in the population.

We have assumed above that ability was distributed independently across agents. This is a relatively strong assumption, however, as one might think that ability is correlated either with origin class or with parental ability.[17]

The first case, in which ability is (positively) correlated with birth class, can be readily accommodated in the model. Letting the probabilities of high ability differ for the two classes by introducing p_0 and p_1 in the equations in (3) changes the critical values of λ and thereby changes the relative sizes of the regions in Figure 2. In terms of the evolution of social structure, the rates at which classes change depend on both p_0 and p_1. When there is only attrition or growth the steady states at 0 and 1 are preserved. When both classes display in and out flows, all the candidate fixed points now have values that depend on p_0 and p_1. Modulo that difference, the system displays the same qualitative behaviour.

The case in which individual ability is correlated with parental ability is slightly more complicated. Studying it precisely demands keeping track of the measure of high ability individuals in each class as they change in time. The proportion of high ability agents in each class is no longer constant (p), but changes from period to period. This implies that the critical λ values will change over time. For large or small values of λ this does not cause particular problems, as either all potentially mobile agents work (and rise) or none do (and fall). While this changes the proportion of high ability agents in the origin classes, the implied movements in critical λ values does not change optimal behaviour. Difficulties in analysing the dynamics arise, however, when λ is near the initial critical values. Changing proportions of high ability agents in the two classes can change critical values such that the optimal behaviour for particular types of agents changes from one period to the next.[18]

[17] See Checchi et al. (1999) for a model in which ability is transmitted with persistence from one generation to the next.

[18] Take the case of perfect correlation, assuming low ability is moderately rare and the approbation motive is low (i.e. $\lambda \leq \Lambda_0 < \lambda_0 < \lambda_1 < \Lambda_1$). All individuals expend low effort. Supposing that initially the measure of high ability is p in both classes, after the first period only high ability remain in the upper class and the share of high ability in the lower class becomes $px/\left(x + (1-p)(1-x)\right) < p$. As $\partial\lambda_0/\partial p, \partial\Lambda_0/\partial p > 0$ the critical λs for the lower class are shifted left. Thus now high effort can become an equilibrium for the high ability poor.

4 The model with relative approbation

To this point the model has excluded any feedbacks from the distribution of approbation across players to the value of approbation to a single individual. In this section we consider the possibility that the approbation gained from social mobility could be related to the size of the upper class.

In the model developed in section 2, approbation scarcity exists since the total measure of approbation allocated across the players is always equal to the prior p. There is only intra-class competition however. In the lower class the total measure of approbation to be allocated is $(1-x)\,p$, each agent who climbs receiving 1, and each agent who stays put receiving $(p - puq_{0,A})\,/\,(1 - puq_{0,A})$. Similarly the total amount of approbation to allocate across upper class members is px, divided among those who fall and those who remain. Thus the competition for the scare approbation resource is circumscribed within each origin class. A global competition for approbation will exist, though, if agents care not about absolute levels of approbation, but rather about their levels relative to the rest of the population. This idea can be formalized by writing the individual's utility function as

$$r\,(z_i) - c(e_i) + \lambda \sum_{\theta<\theta_i} \mu\,(\theta)\,,$$

where $\mu\,(\theta)$ is the measure of agents with approbation equal to θ.[19] This formalization of a global interaction is very classic in the literature on status and relative utility, where players manipulate strategically their position in the distribution of some observable variable, such as the consumption of a positional good (see Frank 1985b, Robson 1992, Hopkins and Kornienko 2004). A similar interaction structure exists here, with effort provision influencing observable mobility, which influences approbation levels and thus one's ranking in the overall distribution of approbation.

The mobility structure remains as described in section 2, and since we have a only four sub-populations, we can summarize the equilibrium approbation distribution in Table 2 as determined by the size, x, of the upper class.

Approbation level θ	Measure $\mu\,(\theta)$
0	$xd\,(1-p)\,(1-q_{1,a})$
$(p - puq_{0,A})\,/\,(1 - puq_{0,A})$	$(1-x)\,(1 - puq_{0,A})$
$p/\,(1 - d\,(1-p)\,(1-q_{1,a}))$	$x\,(1 - d\,(1-p)\,(1-q_{1,a}))$
1	$(1-x)\,puq_{0,A}$

Table 2: The frequency distribution of approbation.

The 4 possible approbation values have the property that $0 < (p - puq_{0,A})\,/\,(1 - puq_{0,A}) \leq p/\,(1 - d\,(1-p)\,(1-q_{1,a})) < 1$ for any $q_{0,A}$ and $q_{1,a}$, with the central equality only when $q_{0,A} = 1 - q_{1,a} = 0$. Thus the measure of players strictly below any value obtains readily.

As the class distribution (characterised by x) changes, the distribution of approbation changes as well. Since the value to an individual arises from his position in this distribution, changes in the distribution can change his incentives. Thus, there is a feedback from the

[19] In this formalization we use a strict inequality, summing over agents whose approbation is $\theta < \theta_i$. The results are not sensitive to the assumption that this is strict.

class distribution to incentives for high effort. In what follows, then, we focus on critical values of x, rather than λ, and the evolution of x over time, holding λ constant.

4.1 Equilibrium strategies

Following the logic of Section 3.1, the incentives towards high effort for the 4 types of players are found to be

$$\begin{aligned}
&\Delta_{0,a}(q) = -c, \\
&\Delta_{1,A}(q) = -c, \\
&\Delta_{0,A}(q) = \lambda u\left(puq_{0,A} - d\left(1-p\right)\left(1-q_{1,a}\right)\right)x + u\left(1+\lambda\left(1-puq_{0,A}\right)\right) - c, \\
&\Delta_{1,a}(q) = -d\lambda\left(1-puq_{0,A} - d\left(1-p\right)\left(1-q_{1,a}\right)\right)x + d\left(1+\lambda\left(1-puq_{0,A}\right)\right) - c.
\end{aligned} \tag{8}$$

An immediate observation is that $\partial\Delta_{1,a}(q)/\partial x \leq 0$, that is, as the upper class grows, its members are less likely to put in high effort.

A major difference between Equation 8 and its corresponding Equation 3 in the model with absolute approbation is that here decisions are not independent across classes. Therefore, solving simultaneously $\Delta_{0,A}(q) = \Delta_{1,a}(q) = 0$ for $q_0, q_1 \in (0,1)$ yields the asymmetric equilibrium

$$\begin{aligned}
q_0 &= \frac{1}{2}\frac{2du\left(1+\lambda\right) - u\lambda xd - c\left(u+d\right)}{\lambda pu^2 d\left(1-x\right)}, \\
q_1 &= \frac{1}{2}\frac{-\lambda xdu\left(1-2d\left(1-p\right)\right) - c\left(u-d\right)}{u\lambda xd^2\left(1-p\right)}.
\end{aligned}$$

Constraining q_0 and q_1 to take their extreme values $\{0,1\}$ defines the critical x-values for symmetric equilibria. They are written

$$\begin{aligned}
x_0 &= \frac{2du\left(1+\lambda\right) - c\left(u+d\right)}{u\lambda d}, \\
X_0 &= \frac{2du\left(1+\lambda\right) - c\left(u+d\right) - 2\lambda pu^2 d}{u\lambda d\left(1-2pu\right)}, \\
x_1 &= c\frac{d-u}{\lambda d\left(1-2d\left(1-p\right)\right)}, \\
X_1 &= c\frac{d-u}{\lambda d},
\end{aligned} \tag{9}$$

where $q_0(x_0) = 0$; $q_0(X_0) = 1$; $q_1(x_1) = 0$; $q_0(X_0) = 1$. Equilibrium behaviour in relation to the size of the upper class, x, is summarized in Proposition 3.

Proposition 3 *Equilibrium is characterized by*

$$\begin{aligned}
q^*_{0,a} &= 0, \\
q^*_{1,A} &= 0, \\
q^*_{0,A} &= \begin{cases} 0 & \text{if } x_0 < x, \\ q_0 & \text{if } X_0 \leq x \leq x_0, \\ 1 & \text{if } x < X_0, \end{cases} \\
q^*_{1,a} &= \begin{cases} 0 & \text{if } x_1 < x, \\ q_1 & \text{if } X_1 \leq x \leq x_1, \\ 1 & \text{if } x < X_1. \end{cases}
\end{aligned}$$

Proof. From Equation 9, $q_0(x_0) = 0$; $q_0(X_0) = 1$; $q_1(x_1) = 0$; $q_0(X_0) = 1$, so the proposition holds if $X_i \le x_i$. For the upper class we have already seen that $\partial\Delta_{1,a}(q)/\partial x \le 0$, thus $q^*_{1,a}$ is weakly decreasing with x, so $X_1 \le x_1$. For high-ability, low-origin agents,

$$\partial q_0/\partial x = \frac{1}{2}\frac{ud\,(\lambda+2) - c\,(u+d)}{\lambda p u^2 d\,(x-1)^2},$$

which changes sign in λ. Check where it changes sign by solving $\partial q_{0,A}/\partial x = 0$ for λ, which yields a unique $\lambda^* = \frac{c}{d} + \frac{c}{u} - 2 > 0$ (since we have assumed that $c > \max(d,u)$). Substituting λ^* into x_0 and X_0 we get we get $x_0 = X_0 = 1$. For $\lambda < \lambda^*$, $X_0 < x_0 < 1$ and for $\lambda^* < \lambda$, $1 < x_0 < X_0$. The proposition follows. ■

To contrast the analysis with the results from Section 3.2, observe that for both classes the measure of high society has a negative impact on the incentives towards high effort: a larger high society makes high society less desirable for both potential entrants and incumbents.

4.2 Evolution of social structure

As in Section 3.2, we model a structure in which an agent's origin class is inherited without error from its parent. Following the logic of that section, the evolution of social structure is described by $x^{t+1} = x^t\{1 + r^+(x^t) - r^-(x^t)\} = T(x^t)$, with $r^+(x) = (1/x - 1)puq^*_{0,A}$ and $r^-(x) = (1-p)d(1-q^*_{1,a})$. Figure 3 shows an illustrative example using the same parameter values as in Figure 1. Low-origin agents work hard when the upper class is small. That group moves smoothly, but quickly to low effort as the upper class grows (over the range $[X_0, x_0]$), and is shown as circles on the left panel. Also, because $d > u$, there is a strategy change for high-origin agents from E to e over $[X_1, x_1]$ (thick black curve on the left panel). These changes in strategy cause changes in the slope of the map on the right panel at the corresponding critical x-values.

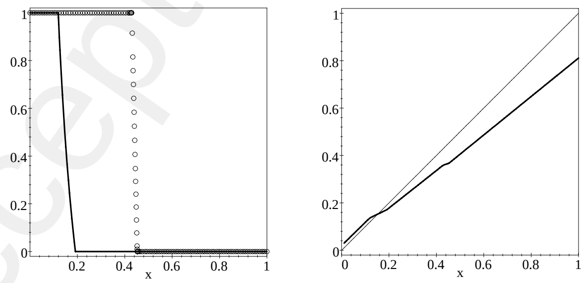

Figure 3: The equilibrium $q^*_{0,A}$ and $q^*_{1,a}$ (left panel) and the map T describing the evolution of the measure of high society for $p = .5$, $u = .2$, $d = .21$, $c = .3$ and $\lambda = .6$ (right panel).

We can completely characterise the steady state social structure. Define $\lambda^* = -1 + c\,(u+d)\,/2du$ to be the value of the approbation motive such that some low-origin agents start expending high effort. The following proposition summarizes the results.

Proposition 4 *Consider two cases, as defined by the ordering of u and d.*

- *Suppose* $u \geq d$. *Then*

$$x^{\infty} = \begin{cases} 0 & \text{if } \lambda \leq \lambda^*, \\ \left\{ \frac{up}{up+d(1-p)}, \frac{2du(1+\lambda)-c(u+d)}{du\lambda(1+2d(1-p))} \right\} & \text{if } \lambda > \lambda^*. \end{cases}$$

- *Suppose* $u < d$. *Then*

$$x^{\infty} = \begin{cases} \{x^{t=0}, X_1\} & \text{if } \lambda \leq \lambda^*, \\ \{x^{t=0}, x_0, X_1, x^*\} & \text{if } \lambda > \lambda^*, \end{cases}$$

with x^* *defined as* $\arg_x\{(1/x-1)puq^*_{0,A} = (1-p)d(1-q^*_{1,a})\}$,*, and discussed below.*

Proof. The proof consists of defining fixed points of the map $T(x)$, and showing that they are stable. $T(x)$ is a continuous mapping of the unit interval onto itself. Therefore a stable fixed point (possibly extreme) always exists. If an interior fixed point exists, it is stable. Differentiate $T(x)$ yields

$$T'(x) = 1 - pu\left(q^*_{0,A} - (1-x)\frac{\partial q^*_{0,A}}{\partial x}\right) - (1-p)\,d\left(1 - q^*_{1,a} - x\frac{\partial q^*_{1,a}}{\partial x}\right).$$

If $q^*_{0,A}$ or $q^*_{1,a}$ belongs to $\{0,1\}$, their derivatives are 0. In either case $T^*(x) \leq 1$. For the interior cases, simple calculation and substitution of the derivatives yields $0 \leq T'(x) \leq 1$. Thus any interior fixed point is stable. If there is no interior fixed point one of the extreme points is stable.

Now we turn to the fixed points, which follow from substituting defined parameter values into $T(x)$. If $u \geq d$ then $x_1 \leq 0$ and $q^*_{1,a} = 0$, so departures from the upper class happen at the rate $r^-(x) = (1-p)d$. There are two cases for the lower class: a) if $\lambda \leq \lambda^*$, $q^*_{0,A} = 0$ and there is no upward motion; the upper class shrinks indefinitely, the lower class grows indefinitely, until, in the limit, $x = 0$; b) if $\lambda > \lambda^*$ then $q^*_{0,A} > 0$, and assuming that $x^{t=0} > 0$, there is positive upward motion over $[0, x_0)$. Since in this case $T(0) > 0$ and $T(1) = 1 - (1-p)d < 1$ there is a unique stable stationary size of the upper class. If the crossing takes place before x_0 then $x^{\infty} = \frac{up}{up+d(1-p)}$, the root also identified in Section 3.2. If the crossing is between x_0 and X_0 then $x^{\infty} = \arg_x\{(1/x-1)puq^*_{0,A} = (1-p)d\} = \frac{2du(1+\lambda)-c(u+d)}{du\lambda(1+2d(1-p))}$.

Suppose now $u < d$. Then $q^*_{1,a} = 1$ over $[0, X_1]$, and $0 < q^*_{1,a} < 1$ over (X_1, x_1). If $\lambda \leq \lambda^*$, then $q^*_{0,A} = 0$ and there is no upward motion. Thus if $x^{t=0} \in [0, X_1]$ society is a *dynasty* since there is neither upward nor downward motion. When $x^{t=0} > X_1$ a proportion $1 - q^*_{1,a}$ of high society is demoted each period. This is compensated by no inflow; society is a *negative meritocracy* and $x^{\infty} = X_1$. (In this case the society approaches a dynasty.) Thus in that parameter region $x^{\infty} \in [0, X_1]$. Finally if $\lambda > \lambda^*$ several situations are possible, depending on the ordering of critical x-values. With similar arguments about flows, it is possible to show the following. When $x_0 < X_1$, any $x \in [x_0, X_1]$ is stationary and society is either a *dynasty* if $x^{t=0} \in [x_0, X_1]$ or a (*positive* or *negative*) *meritocracy* with $x^{\infty} \in \{x_0, X_1\}$ if $x^{t=0} \notin [x_0, X_1]$. By contrast, when $x_0 \geq X_1$, there is never the conjunction of a purely industrious upper class and a purely lazy lower class, thus a unique x^{∞} with meritocratic features will obtain, with $x^* = \arg_x\{(1/x-1)puq^*_{0,A} = (1-p)d(1-q^*_{1,a})\}$. ■

Depending on the relative importance of the possibility of falling against the potential of a promotion, different long run situations arise. With relatively easier upwards mobility

($u > d$) society has only lazy rich. The poor (or a fraction of them) will be industrious if the approbation motive is strong enough ($\lambda > \lambda^*$), which results in a *meritocracy.* But as soon as a high-ability poor has risen, he turns into a passive rich, as do his offspring, eventually leading the family back to the lower class. Even if high effort prevails over the whole unit interval for the high ability poor, high society is never bigger than $\frac{up}{up+d(1-p)}$, the root already identified in Section 3.2.

By contrast, with relatively harder upwards mobility ($u < d$) society has (at least a fraction of) industrious rich, and the lower is the approbation motive the more numerous are the industrious rich; for them λ has a negative effect on the willingness to give high effort. This is not so for the poor, who demand that $\lambda > \lambda^*$ to start providing high effort. Hence below λ^* no poor rises, while below X_1 no rich falls. This implies a dynasty on the interval $[0, X_1]$. On the right side of X_1 there is (at least partial) attrition in the upper class; thus in the absence of lower class promotion *negative meritocracy* dynamics leads society to X_1. Finally when $\lambda > \lambda^*$ the conjunction of (at least partial) lower and upper class effort gives rise most of the time to a *meritocracy* with a unique stationary state (when $x_0 \geq X_1$), though it is also possible (when $x_0 < X_1$) that a *dynasty* establishes itself, with no extreme situation part of it.

When contrasted with Section 3.2, here the approbation motive affects the rich and the poor differently: there is always a value of λ above which the poor will play E, while (provided $u < d$) the provision of E by the rich, by contrast, demands a low enough approbation motive. In general a society whose members care a lot for relative approbation will display meritocratic features, while one in which it is not so will tend to be dynastic.

4.3 Welfare

Consider total utility as the measure of social welfare. The low-ability poor and high-ability rich are immobile whatever they do, so both the private and social optimum for them is to expend low effort, in both models.

When absolute approbation matters as in Section 3.2, upper class low-ability and lower class high-ability face independent problems which can be solved as such. Defining s_1 as the socially optimal effort level for the low-ability upper class, social optimality requires the maximization of $s_1(1 + \lambda p/[1 - d(1-p)(1-s_1)] - c) + (1-s_1)(1-d)(1 + \lambda p/[1 - d(1-p)(1-s_1)])$. The first order condition yields two conjugate solutions, of which one is always negative. The second order condition for a maximum holds at

$$s_1 = \frac{1}{c-d}\frac{-(c-d)(1-(1-p)d) + p\sqrt{d\lambda(c-d)}}{(1-p)d}.$$

Direct computation shows that the difference between socially and privately optimal behaviour is

$$s_1 - q^*_{1,a} = p\frac{\sqrt{d\lambda(c-d)} - d\lambda}{(c-d)d(1-p)}.$$

If $\lambda > \lambda^* = \frac{c-d}{d}$, $s_1 < q^*_{1,a}$ and there is over-provision of high effort (if $q^*_{1,a} > 0$). From the definitions in section 3.1, (Equations 4 and 5), $\lambda^* < \Lambda_1 < \lambda_1$. High effort is provided by some members of this sub-population if $\lambda > \Lambda_1$ so we can conclude that there will be over-provision of effort if λ falls between Λ_1 and λ_1, but never under-provision.

Similarly for the lower class, social optimality requires the maximization of $s_0(u\,(1+\lambda)+(1-u)\,\lambda\,(p-pus_0)\,/\,(1-pus_0)-c)+(1-s_0)\,\lambda\,(p-pus_0)\,/\,(1-pus_0)$. This is a convex function of s_0; thus the socially optimal s_0 is either 0 when

$$\lambda < \lambda^* = \frac{c-u}{u\,(1-p)^2}\,(1-pu)\,,$$

or 1 otherwise. From Equations 4 and 5, the critical values Λ_0 and λ_0 are both less than λ^* ($\Lambda_0 \leq \lambda_0 \leq \lambda^*$). Thus over-provision of effort can happen for $\Lambda_0 < \lambda < \lambda^*$ and does happen for $\lambda_0 < \lambda < \lambda^*$ for lower class agents. Approbation-seeking results in a waste, as in most models in which agents engage in a comparable form of social contest.

Consider now the case of relative approbation seeking. Total welfare is written as the sum of the contributions of all 4 types of players (though both the social and private optima for the talentless poor and the smart rich are to play e, the measure of talentless rich and smart poor expending E affects their contribution to global welfare). A quadratic form obtains, which is strictly concave in s_0 and s_1. The first order conditions are thus sufficient to characterize the unique socially optimal proportion of agents in each class who should expend E, which are

$$\begin{aligned} s_0 &= \frac{1}{2}\frac{-xu\lambda - c + u + u\lambda}{(1-x)\,u^2\lambda p}, \\ s_1 &= \frac{1}{2}\frac{2\lambda xd^2p + xp\lambda^2 d - \lambda xdp + c + \lambda xd - d - 2\lambda xd^2}{(p-1)\,\lambda xd^2}. \end{aligned}$$

When these values lie in the unit square they characterize the unique socially optimal effort levels in each of the relevant groups.[20] Computing the difference between social and private optimality yields

$$\begin{aligned} s_0 - q^*_{0,A} &= \frac{1}{2u}\frac{c-d-d\lambda}{\lambda pd\,(1-x)}, \\ s_1 - q^*_{1,A} &= \frac{1}{2d}\frac{u\lambda xp\,(1-\lambda)+u-c}{u\lambda x\,(1-p)}. \end{aligned}$$

Thus for lower class, the possibility of under-provision arises as soon as $\lambda < (c-d)\,/d$ (recall $c > d$), that is, under provision is possible and associated with small values of the approbation motive λ. In the upper class under provision demands that $\lambda\,(1-\lambda) > \frac{c-u}{uxp}$, that is, the approbation motive λ should be neither too small nor too large.

In the case of relative approbation seeking, a waste does not necessarily obtain; the quest for a higher quantile position can yield a suboptimal outcome in the form of a shortage rather than an over-provision of effort.

5 Conclusion

Though being considered a talented person does not really make one economically better off, it is something we tend to value positively. What I wear, which car I drive and where I spend

[20] It is straightforward to check that for the values $(p, \lambda, u, d, c) = (0.5, 0.5, 0.79, 0.8, 0.81)$, over the interval $x \in (0.759, 0.949)$ the optimum is strictly within the unit square, and there is under-provision of effort by the upper class. Given the continuity of the functions, we can conclude that for a non-degenerate area of the parameter space, under-provision of effort is the equilibrium.

my week-ends can all be considered correlated with ability, but probably a more revealing signal is my social mobility, as measured by my educational or professional achievement relative to that of my parents. The model developed here is about how individual incentives affect the evolution of social structure and about how the system of social mobility self-regulates.

In the model, an inherent concern for social approbation (a concern for status, if status is determined by ranking on the scale of approbation) is transformed into an instrumental concern for social class. It is not simply class per se, however, in the sense in which approbation arises from class in an aristocracy. Rather, it is a concern for what we have termed "destination class", that is, the class an agent is in after mobility has taken place. Comparing origin and destination class is what permits public inference about the trait on which approbation is determined. Thus it is mobility between classes (or lack of it) rather than the class into which one is born that matters for our agents. Because mobility is a source of well-being to individuals and they actively seek it, social structure changes over time and we study the types of organizations that form.

One interesting result that emerges is that in spite of the fact that social mobility is determined purely on meritocratic principles, society can organize into an aristocracy. One of the over-arching concerns in empirical studies of social mobility is the extent to which a society is "meritocratic".[21] This debate is conducted by examining different empirical measures of mobility between social classes. But our results suggest that the issue may be more subtle. Is a society meritocratic if it has the right kind of mobility, or is it meritocratic if it has the right kind of incentives? If the latter, it may not be enough to look at mobility in order to conclude that a society is or is not a meritocracy. When the trait on which social approbation is granted is relatively common, then a society with meritocratic incentives will have as its characteristic social "mobility regime" a dynasty. All children end up in the social class into which they were born. Indeed, because everyone is presumed to be of high ability, if a low origin agent rises this has little effect on people's estimates of his ability, owing to the initial presumption that he was high ability. Thus the effort needed to rise in status produces little value in terms of approbation. On the other hand, if a high origin agent falls, this is an unambiguous signal that he is of low ability. Thus the effort needed to stay in the upper class is worth it. These effects combine to produce a frozen inter-generational social structure.

In the second part of the paper, we modify how approbation enters the utility function to capture a competition for rank in the approbation distribution. Indirectly, this introduces a feedback between the social structure on the one hand, and effort and social mobility on the other. Effort levels and social structure are jointly endogenized. In contrast with the simple model, here the weight of approbation in the utility function has a different effect on stable structures. In this case, dynasties can only emerge when concern for approbation is weak, whereas in the previous case they emerged for intermediate levels of concern. A second effect of the introduction of competition for approbation rank has to do with welfare: without the rank competition, neither class ever under-provides effort, but can over-provide effort for some parameter values. With rank competition, however, under-provision of effort by either class (or both of them) is possible.

[21]See the debate between Saunders, who finds the UK to be highly meritocratic, and Breen and Goldthorpe (1999), who do not (and our discussion on p. 3).

Looking slightly beyond the model permits an interesting suggestion about how mobility could change over time. One can imagine that from time to time new approbation traits emerge. Suddenly something new, ability in mathematics for example, becomes highly esteemed. One would expect that initially esteem is given to relatively rare traits. When this is the case, social mobility itself demonstrates meritocratic features. When low ability is presumed (that is when the approbation trait is rare), then a rise by a low origin agent is an unambiguous signal of high ability, while the fall of a high origin agent will not change very much the public opinion about his ability. Thus low origin agents have strong incentives to work hard, but high origin agents have only very weak incentives. High ability, industrious poor advance; low ability, lazy rich fall, and the mobility "looks meritocratic". But when possession of a trait provides utility, a rational response is to invest. We often see parents forcing children to invest (typically in education) so they will acquire these rare traits for which approbation is granted. Rarity falls, and mobility will be reduced, possibly disappearing altogether as the trait becomes too common and society moves into a frozen dynastic structure. This changes not only the nature of mobility, but also which part of the population works hard. Initially it is the striving, high ability poor, but eventually it is the defensive, low ability rich. If upper class income were determined not only by the size of the class but also by the ability of those earning it, social welfare could increase if high-ability agents populated the upper class. Thus a shift from an uncommon to common approbation trait, and the consequent move from a meritocracy to a dynastic structure, would have negative implications for social welfare.

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