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Collusion with private and aggregate information

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Veröffentlichungsversion / Published Version
Arbeitspapier / working paper

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Empfohlene Zitierung / Suggested Citation:
Jin, J. Y. (1999). *Collusion with private and aggregate information.* (Discussion Papers / Wissenschaftszentrum Berlin für Sozialforschung, Forschungsschwerpunkt Marktprozeß und Unternehmensentwicklung, Abteilung Wettbewerbsfähigkeit und industrieller Wandel, 99-10). Berlin: Wissenschaftszentrum Berlin für Sozialforschung gGmbH. https://nbn-resolving.org/urn:nbn:de:0168-ssoar-194508

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discussion papers

FS IV 99 - 10

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September 1999

ISSN Nr. 0722 - 6748

Forschungsschwerpunkt Marktprozeß und Unternehmensentwicklung

Research Area Market Processes and Corporate Development

Zitierweise/Citation:

Yim Y. Yin, **Collusion with Private and Aggregate Information,**
Discussion Paper FS IV 99 - 10, Wissenschaftszentrum Berlin, 1999.

Wissenschaftszentrum Berlin für Sozialforschung gGmbH,
Reichpietschufer 50, 10785 Berlin, Tel. (030) 2 54 91 - 0

ABSTRACT

Collusion with Private and Aggregate Information

by Jim Y. Jin*

This paper considers three linear asymmetric oligopoly models with (i) a representative consumer, (ii) horizontal differentiation and (iii) vertical differentiation. We show that firms could maximize the joint-profit only based on private and aggregate information. They can choose the "correct" colluding prices without knowing the demand or profit function. The collusive outcome is a natural focal point despite firms are asymmetric. Collusion can be incentive compatible even though individual actions (prices) are not observed.

ZUSAMMENFASSUNG

Absprache durch private und gemeinschaftliche Information

Der Beitrag untersucht drei linear asymmetrische Oligopol-Modelle mit (i) einem repräsentativen Verbraucher, (ii) horizontaler Differenzierung und (iii) vertikaler Differenzierung. Es wird gezeigt, daß Firmen in der Lage sind, den Gesamtprofit allein auf der Grundlage privater und gemeinschaftlicher Information zu maximieren. Sie können zur „richtigen" Absprache des Preises gelangen, ohne die Nachfrage- oder Gewinn-Funktion zu kennen. Die Absprache stellt einen natürlichen Gleichgewichtspunkt dar, ungeachtet asymmetrischer Verhältnisse. Die Absprache kann anreizkompatibel sein, auch wenn individuelle Aktionen (Preise) nicht beobachtet werden.

* I thank R. Amir, H. Mewis, M. Tröge and B. Wernerfelt for their helpful comments. The responsibility for remaining errors is solely mine.

1. Introduction

To achieve the joint-profit-maximization, firms should know how to choose appropriate prices and have incentives to do so. This often requires (unrealistically) complete and perfect information. This paper shows that, in three basic linear oligopoly models, firms can maximize the joint-profit only based on private and aggregate information, much less than one expects according to existing theories.

The difficulty to collude under imperfect information has been long realized. Stigler (1964) argued that observing each other's actions is essential for effective punishment. Relying on perfect monitoring Friedman (1971) established the first rigorous collusion equilibrium in a supergame. In Green and Porter (1984, tacit collusion with unobservable actions (quantities) can only be achieved in a homogeneous Cournot oligopoly, and is often interrupted by price wars.

Incomplete information also prevents firms from choosing ”correct” collusive prices even if they want to do so. Lack of complete information often stems from the firm heterogeneity, which alone makes collusion difficult. Asymmetric firms cannot easily agree upon a focal point because the joint profit maximization may not divide the pie fairly. As pointed out by Tirole (1988), collusion becomes more questionable when firms "*offer differentiated products (differentiated according to quality, location, distribution channels, etc.). It is often felt that heterogeneity in both costs and products may make coordination on a given price difficult*" (p. 242).

Early collusion models normally assume homogeneous products and quantity competition. The later extension into differentiated products with a representative consumer includes Deneckere (1983), d'Aspremont et al (1983), Wernerfelt (1989) and

Rothschild (1992). Furthermore, collusion with horizontally differentiated products was examined by Albaek and Lambertini (1998), Chang (1991), Jehiel (1992) and Ross (1992). These models are usually symmetric and do not focus on the difficulty of collusion among asymmetric firms. Donsimoni (1985), Bae (1987) and Verboven (1997) studied collusion with heterogeneous firms, yet under complete and perfect information. An exception is Verboven's (1998) horizontal differentiation model with three firms, where two firms at ends cannot observe each other's quantities. In his model firms have to soften their punishments, and thus are less likely to achieve perfect collusion. Recently, in homogeneous Cournot oligopoly Rothschild (1999) showed that collusion is likely hindered by asymmetric costs.

In the collusion literature firm asymmetry and incomplete/imperfect information are usually not addressed simultaneously. The heterogeneity in product qualities and costs make it difficult for firms to agree upon a focal point. In addition, if firms only know their own product characteristics, they are usually unable to choose appropriate prices. Moreover, if actions are unobservable, firms cannot effectively punish defection and collusion becomes unsustainable. These difficulties all together seem too big and complex for firms to overcome. A common sense would probably dismiss the real possibility of perfect collusion by large.

It will be argued in this paper, that in three basic linear oligopoly models, firms can maximize the joint-profit only based private and aggregate information. The three asymmetric models with differentiated products are characterized by (i) a representative consumer, (ii) horizontal differentiation, (iii) vertical differentiation. The private information merely covers a firm's product quality and cost, and the aggregate information is about social welfare and consumer surplus only.

The paper is organized as follows. In the next section, we introduce three models. Section 3 shows that, firms can chooses perfect collusive prices only based on private information. The incentive compatibility given aggregate information is proved in section 4. Section 5 summarizes and discusses the policy implication.

2. Model

(i) *Model I: Differentiated Products with a Representative Consumer*:

This model extends those considered by Deneckere (1983), d'Aspremont et al (1983), Donsimoni (1985), Bae (1987), Wernerfelt and Rothschild (1989) and Verboven (1997). There are n firms. Each of them produces one good at a constant marginal cost in every period. Firm i's price, output and marginal cost in period t are denoted by p_{it}, x_{it}, and c_{it}. Denote the price and output vectors by $\mathbf{p}_t$ and $\mathbf{x}_t$. In addition to the n products, there is a numeraire good x_{0t} sold at a constant price normalized to 1.

A representative consumer has a utility function $U_t = x_{0t} + \mathbf{a}_t \cdot \mathbf{x}_t - 0.5\mathbf{x}_t' B_t \mathbf{x}_t$, where $\mathbf{a}_t$ is an n×1 vector and B_t is a n×n matrix. Both $\mathbf{a}_t$ and B_t vary in each period. Given the price vector $\mathbf{p}_t$, her income w_t and the budget constraint $x_{0t} + \mathbf{p}_t \cdot \mathbf{x}_t \leq w_t$, the representative consumer chooses x_{0t} and $\mathbf{x}_t$ to maximize her utility. The utility is strictly concave in $\mathbf{x}_t$, so B_t is positive definite. Its elements may have different signs as well as different values, implying a mixture of substitute and complementary goods. Assume that w_t is sufficiently high in each period so that an interior solution for optimal consumption always exists.

No firm knows the entire utility function. Each firm i only knows a_i, the marginal utility of its product when no goods are sold. It depends only on the quality of the product, and should be known by the producer. No firm knows anything about the B_t.

The first-order condition for utility maximization is $p_{it} - a_{it} - \sum_{j=1}^{n} b_{ijt} x_{jt} = 0$ for all i. From this condition we get the demand function in price competition. Since B_t is positive definite, its inverse B_t^{-1} exists. Denote its element by β_{ijt}, and the elements of $B_t^{-1}\mathbf{a}_t$ by α_{it}. We can write the demand function in price competition as:

$$x_{it}(\mathbf{p}_t) = \alpha_{it} - \sum_{j=1}^{n} \beta_{ijt} p_{jt} \qquad (1)$$

Let c_{it} be firm i's constant marginal cost in period t, which varies over time, and is known only by firm i. We assume that, in each period if all firms set prices equal to their marginal costs, every firm can still sell something, i.e., $x_i(\mathbf{c}_t) > 0$ for all i, where $\mathbf{c}_t$ is the n-firm cost vector. This assumption ensures that all firms are secured players. No one can be driven out of the market by others, even if the other firms play minimax strategies with marginal cost pricing.

We further assume $a_{it} > c_{it}$ for all i and t. If this condition does not hold, firm i would be unable to survive even if no other firm exists. Given the demand function (1) and its marginal cost c_{it}, firm i's profit function in period t is $(\alpha_{it} - \sum_{j=1}^{n} \beta_{ijt} p_{jt})(p_{it} - c_{it})$. The joint profit in period t is:

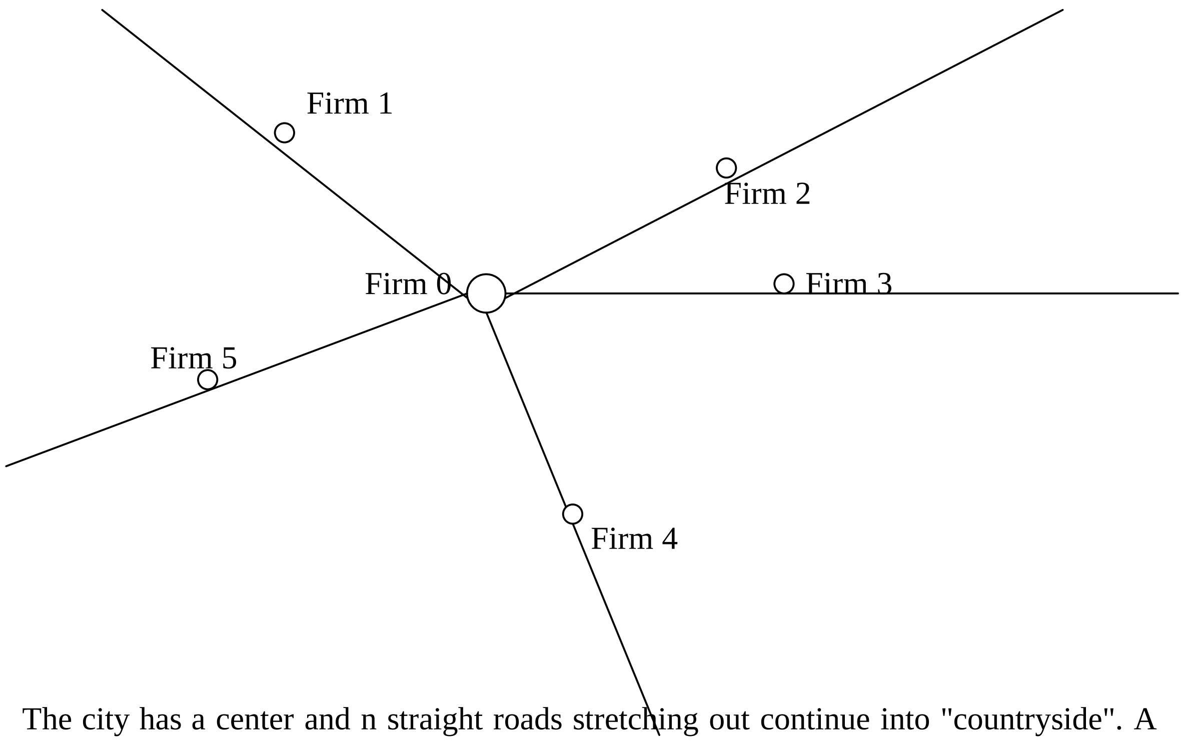
$$\pi_t = \sum_{i=1}^{n} [(\alpha_{it} - \sum_{j=1}^{n} \beta_{ijt} p_{jt})(p_{it} - c_{it})] \qquad (2)$$

Except its own cost, no firm knows any parameter among α_{it}'s and β_{ijt}'s in (2). It seems very unlikely for any firm to choose a joint-profit-maximizing price.

(ii) Model II: Horizontal Differentiation/A Star-City:

Hotelling's model with horizontal differentiation has been used in the collusion literature by Albaek and Lambertini (1998), Chang (1991), Jehiel (1992) and Ross (1992). Verboven (1998) extended their duopoly model to triopoly and assumed that two firms at the ends do not observe each other's actions. Our model further extends his model to oligopoly. Instead of a straight line, we consider a star-city model as shown in the graph.

Fig. 1 Star-City



The city has a center and n straight roads stretching out continue into "countryside". A shopping mall (firm 0) stands at the center and in each of the n roads stands a shopping plazas (firm i = 1, … n) with a unit distance from the center. Consumers reside along n roads with uniform density 1. In period t firm i offers its service with a quality v_{it} which is known only by this firm. A consumer obtains such a utility v_{it} purchasing its product, and incurs a transportation cost τ per unit of distance in addition to the price p_{it}. She chooses the product with the highest surplus provided it is positive. Otherwise she does not buy. Different from the notation in the previous model, product x_{0t} is offered by an oligopoly firm, not a numeraire good. If all n + 1 firms sell something and all consumers between the central mall and plazas purchase, we have the following demand function:

$$x_{0t} = \frac{n}{2\tau}(\tau + v_{0t} - p_{0t}) - \sum_{i=1}^{n} \frac{(v_{it} - p_{it})}{2\tau}, \qquad x_{it} = \frac{\tau + 3v_{it} - 3p_{it} - v_{0t} + p_{0t}}{2\tau} \qquad (3)$$

Each firm i knows τ and its cost c_{it} which varies over time, but $c_{it} < v_{it}$. Similar to model I, we want to make sure that every firm can sell something when all prices are equal to marginal costs. To ensure this we assume that for all i and t:

$$|v_{0t} - c_{0t} - v_{it} + c_{it}| < \tau \qquad (4)$$

To ensure that the joint-profit maximization yields an interior solution, we assume that v_{it}'s are sufficiently high comparing to costs such that

$$v_{0t} + v_{it} > 5\tau + c_{0t} + c_{i\,t} \qquad (5)$$

Given the demand function (3) and marginal costs, the total profit is:

$$\pi_t = \frac{(p_{0t}-c_{0t})}{2\tau}[n(\tau + v_{0t} - p_{0t}) - \sum_{i=1}^{n} (v_{it} - p_{it})] + \sum_{i=1}^{n} \frac{(p_{it}-c_{it})}{2\tau}(\tau + 3v_{it} - 3p_{it} - v_{0t} + p_{0t}) \qquad (6)$$

Knowing τ, v_{it} and c_{it} only, firm i does not seem capable of setting the joint-profit maximizing price even if it wants.

(iii) Model III: Vertically differentiated products:

The model is based on Gabszewicz and Thisse (1979) and Shaked and Sutton (1982). To my knowledge, collusion with vertical differentiation has not been studied in the literature. Here we extend the well-known vertically differentiated duopoly model to oligopoly. There are n firms selling n products with different qualities and constant marginal costs. They are ranked according to their distinct qualities. Let q_{it} be firm i's quality, which varies overtime and is known only by firm i. Without loss of generality, we assume $q_{i+1,t} > q_{it}$ for all i. An infinite number of consumers indexed by θ are uniformly distributed within [0,1]. Every consumer θ chooses the highest $\theta q_{it} - p_{it}$ to maximize her surplus by purchasing at most one product. No purchase occurs if the surplus is negative. When every firm sells something, we have:

$$x_{1t} = \frac{p_{2t} - p_{1t}}{q_{2t} - q_{1t}} - \frac{p_{1t}}{q_{1t}} \qquad x_{nt} = 1 - \frac{p_{nt} - p_{n-1t}}{q_{nt} - q_{n-1t}},$$

$$x_{it} = \frac{p_{i+1t} - p_{it}}{q_{i+1t} - q_{it}} - \frac{p_{it} - p_{i-1t}}{q_{it} - q_{i-1t}} \qquad \text{for } 1 < i < n \qquad (7)$$

Firms' marginal costs depend on qualities according to a function c(q), which is unknown to firms. However, the function satisfies the following properties:

$$c(0) = 0, \qquad c'(q) > 0, \qquad c''(q) > 0$$

$$c_{it} < q_{it}, \qquad q_{it} - c_{it} < q_{i+1t} - c_{i+1t} \qquad (8)$$

This first row is the standard assumption. If the inequality in the second row is violated, firm i cannot sell anything unless it incurs a loss. The right inequality is necessary to ensure that every firm can sell something under marginal cost pricing. If it is violated, no one buys from firm i + 1 when $p_{it} = c_{it}$. Given the demand function (7), the total profit in period t is:

$$\pi_t = \left(\frac{p_{2t}-p_{1t}}{q_{2t}-q_{1t}} - \frac{p_{1t}}{q_{1t}}\right)(p_{1t} - c_{1t}) + \left(1 - \frac{p_{nt}-p_{n-1t}}{q_{nt}-q_{n-1t}}\right)(p_{nt}-c_{nt})$$

$$+ \sum_{i=2}^{n-1}(p_{it} - c_{it}) \left(\frac{p_{i+1t}-p_{it}}{q_{i+1t}-q_{it}} - \frac{p_{it}-p_{i-1t}}{q_{it}-q_{i-1t}}\right) \qquad (9)$$

Knowing only q_{it} and c_{it}, setting a joint-profit maximizing price seems unlikely. In all three oligopoly models, we assume that for every firm i: $\sum_{k=t}^{\infty}\delta_i^{k-t}E(\pi_{ik}^c) > E(\pi_{it}^d) + \sum_{k=t+1}^{\infty}\delta_i^{k-t}E(\pi_{ik}^n)$, where $E(\pi_{ik}^c)$ is firm i's expected collusive profit, $E(\pi_{it}^d)$ is its expected defecting payoff, $E(\pi_{ik}^n)$ is the expected payoff when firms play a non-cooperative game in period k, δ_i is firm i's discount factor. This assumption ensures that if information is perfect and complete, every firm prefers collusion to defection. Thus, we can focus on three problems: how to find (a) collusive prices, (b) an agreeable focal point and (c) defection behavior, given very limited information.

Besides the private information about firms' own product characteristics above, we allow only aggregate information available for firms. However, any aggregate information must come from collection of private information. We assume that there is a public institute collecting all information firms have, i.e., $\mathbf{a}_t$, $\mathbf{c}_t$ in model I, $\mathbf{v}_t$, $\mathbf{c}_t$ and τ in model II, and $\mathbf{q}_t$ and $\mathbf{c}_t$ in model III. Every firm must know its own price and output, so the institute also collects $\mathbf{p}_t$ and $\mathbf{x}_t$ in each period.

No firm's individual data or its behavior will be observed by rivals. The institute can only reveal aggregate information such as social welfare and consumer surplus, based on the information it collects from firms. Since the aggregate information does not reveal any firm's individual behavior, it seems unlikely that firms can use it to detect any defection and monitor collusion effectively.

3. Pricing

Although maximizing the joint-profit looks very difficult given our assumptions, the solution is actually quite simple.

Proposition 1: In oligopoly models I – III, the joint-profit maximizing prices are:

$$p^c_{it} = 0.5(a_{it} + c_{it}) \tag{10}$$

$$p^c_{0t} = 0.5(v_{0t} + c_{0t}) + \tau, \qquad p^c_{it} = 0.5(v_{it} + c_{it} + \tau) \tag{11}$$

$$p^c_{it} = 0.5(q_{it} + c_{it}) \tag{12}$$

Proof: see Appendix A.

A major challenge for asymmetric firms to collude is how to find a focal point. The price strategies (10) – (12) offer a plausible solution for the following reasons.

1. *Pareto efficient*: The total profit is maximized ex post.

2. *Easy to implement*: Strategies (10) - (12) require very limited information.

3. *Rationally irrational*: The strategies are very close to the monopoly prices when no other firm exists. In model I, firm i's monopoly demand function is $(a_{it} - p_{it})/b_{ii}$, and its monopoly price would be $0.5(a_{it} + c_{it})$, identical to (10). In model II, firm 0's monopoly demand function is $n(v_{0t} - p_{0t})/\tau$, and the monopoly price is $0.5(v_{0t} + c_{0t})$, less than (11) by $\tau/2$. Any other firm i's monopoly demand function is $2(v_{it} - p_{it})/\tau$, if $p_{it} \geq v_{it} - \tau$, and $n(v_{it} - p_{it})/\tau - n + 2$ otherwise. Conditions (4) and (5) imply $v_{it} > c_{it} + 2\tau$, and hence the optimal price would be $0.5(v_{it} + c_{it} + \tau) – \tau/n$, less than (11) by τ/n. In model III, firm's

monopoly demand function is 1 - p_{it}/q_{it}, and the monopoly price would be $0.5(q_{it} + c_{it})$, identical to (12).

4. *Proportional reduction*: The outcome requires "fair" sacrifice because every firm's output falls to the half of its output level when all prices are set equal to marginal costs. Substituting strategy (10) into the demand function (1), we get collusive output $\mathbf{x}_t(\mathbf{p}_t^c)$ = $B_t^{-1}(\mathbf{a}_t - 0.5(\mathbf{a}_t+\mathbf{c}_t)) = 0.5\, B_t^{-1}(\mathbf{a}_t - \mathbf{c}_t) = 0.5\mathbf{x}_t(\mathbf{c}_t)$. Similarly, plugging (11), (12) into (4), (7) respectively, every firm's output is exactly the half of x_{it} ($\mathbf{c}_t$). Therefore, under the joint-profit maximization every firm retains exactly its market share when all firms try to maximize their market shares.

5. *Justifiable territory*: Suppose that all firms except firm i set prices equal to marginal costs, firm i's optimal price should be the mid-point between its marginal cost and the zero-quantity price. Then, its output must be exactly the half of its marginal cost pricing output, which is just the collusive one. Hence, every firm's quantity under the joint-profit maximization is equal to its optimal output given other firms' minmax strategies. This territory is fully justified by its strength to defend it.

6. *Fair rewards*: Given the quantity allocation, the distribution of profit is determined by firms' profit margins. The collusive profit margins completely depend on firms' own characteristics. In model I, $p_{it} - c_{it} = 0.5(a_{it} - c_{it})$. In model II, this value is $0.5(v_{it} - c_{it}) + \tau$ for i = 0, and + 0.5τ otherwise. In model III, it is $0.5(q_{it} - c_{it})$. All firms earn collusive profits according to own merits, not free riding on others.

Schmalensee (1987) discussed four possible collusion schemes: The first, a side payment, is illegal; the second, constant market shares, is difficult to maintain under uncertainty; the third, market division is often not practical due to arbitrage. The last, equal proportional reduction in outputs, fits our results quite well. Regarding the proportional reduction in quantity competition, Schmalensee pointed out that, "when cost differ, however, this point has no special attraction" (357) since it does not maximize the joint profit. Also, it is not achievable in quantity competition with limited information. In our three models of price competition, the ”proportional reduction” has obvious advantages and attractions to be the focal point.

7. The last reason is easy monitoring, which will be examined in the next section.

4. Monitoring

As we assumed earlier, a public institute collects information about $\mathbf{c}_t$, $\mathbf{p}_t$ and $\mathbf{x}_t$ in each period, also, $\mathbf{a}_t$ in model I, $\mathbf{v}_t$ and τ in model II, and $\mathbf{q}_t$ in model III. Given this information, the institute can calculate social welfare and consumer surplus. In model I, for instance, as $\mathbf{p}_t = \mathbf{a}_t - B_t\mathbf{x}_t$, social welfare $\mathbf{a}_t\mathbf{x}_t - 0.5\mathbf{x}_tB_t\mathbf{x}_t = (0.5\mathbf{a}_t + 0.5\mathbf{p}_t - \mathbf{c}_t)\mathbf{x}_t$, consumer surplus $\mathbf{a}_t\mathbf{x}_t - 0.5\mathbf{x}_tB_t\mathbf{x}_t - \mathbf{p}_t\mathbf{x}_t = 0.5(\mathbf{a}_t - \mathbf{p}_t)\mathbf{x}_t$. we allow the trade association to access the aggregate information such as social welfare and consumer surplus.

Then, monitoring goes as follows. In every period, if social welfare is equal to three times of consumer surplus in model I and III, or plus $n\tau$ in model II, collusion continues; if the equality does not holds, collusion stops immediately and firms play a non-cooperative game forever. We will prove that, if all firms follow (10) - (12), the collusion condition holds; whenever any price is lower than (10) – (12), the condition breaks down. We rule out any deviation with prices higher than the joint-profit maximization level, because it does not make sense. As we assumed that every firm is worse off in a non-cooperative game, colluding is incentive compatible.

Proposition 2: The joint-profit maximization is incentive compatible if collusion stops whenever SW = 3CS (+ $n\tau$) breaks down in model I, III (II).

Proof: see Appendix B.

Every economic model is fictional somehow. We do not argue that the information assumed here is always available in a real market, and firms do collude in this way. The paper nevertheless points such a possibility that seemingly impossible collusion can be achieved with only private and aggregate information. On the other hand, our result reveals that, social welfare is close to 3 times of consumer surplus under perfect collusion. Such an observation can be used by antitrust authorities to identify collusive behavior. Since this relation holds under perfect collusion regardless of how it is achieved, checking such a relation might be useful for antitrust authorities.

5. Conclusion

This paper shows the possibility of joint-profit maximization with only private and aggregated information. We studied three commonly used oligopoly models with product differentiation. The perfect collusion described in the paper is surely not robust for various modifications. However, based on the similar principle firms may design more practical mechanisms to achieve collusive outcomes. One should notice that, perfect collusion does not occur only in the cases we just saw. Given different market structures, we can give examples where other types of information can also lead to similar outcomes. The purpose of the paper is to show that even with very limited information perfect collusion is not so unthinkable as we thought. The particular point of this model is to demonstrate how aggregate information can be collusion conducive.

The studies on collusion have mainly focused on the potential dangers of individual firm-specific information. Antitrust policy rarely concerns with the availability of aggregate information. This paper sends a warning signal.

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Appendix A:

In model I, the total profit, $(\mathbf{p}_t-\mathbf{c}_t)B^{-1}(\mathbf{a}_t-\mathbf{p}_t)$, is strictly concave in $\mathbf{p}_t$, hence reaches its maximum if the first-order condition $B^{-1}(\mathbf{a}_t+\mathbf{c}_t) - 2B^{-1}\mathbf{p}_t = \mathbf{0}$ holds. The solution is $\mathbf{p}_t = 0.5(\mathbf{a}_t+\mathbf{c}_t)$ i.e., (10). Since $a_{it} > c_{it}$ and $x_i(\mathbf{c}_t) > 0$, every firm i makes a positive profit.

In model II, the first-order condition to maximize (6) is: $n(\tau + v_{0t} - 2p_{0t} + c_{0t}) + \sum_{i=1}^{n}(2p_{it} - v_{it} - c_{it}) = 0$ and $2p_{0t} - v_{0t} - c_{0t} + \tau + 3v_{it} - 6p_{it} + 3c_{it} = 0$, for $i > 0$. It is easy to verify that they are satisfied under (11).

The second-order derivatives are: $\partial^2\pi/\partial p_0^2 = -3/\tau$, $\partial^2\pi/\partial p_i^2 = -n/\tau$ for $i > 0$, $\partial^2\pi/\partial p_i\partial p_0 = 1/\tau$, $\partial^2\pi/\partial p_i\partial p_j = 0$ for $i, j \neq 0$ and $i \neq j$. As the sum of every row of matrix $\partial^2\pi/\partial \mathbf{p}^2$ is negative, it has a dominant diagonal, and must be negative definite (McKenzie 1960, Theorem 2). Thus π is strictly concave, (11) leads to the unique maximum.

Substituting (11) into (3), we get $x^c_{0t} = 0.25[n(\tau + v_{0t} - c_{0t}) - \sum_{i=1}^{n}(v_{it}-c_{it})]/\tau$, and $x^c_{it} = 0.25(\tau + 3v_{it} - 3c_{it} - v_{0t} + c_{0t})/\tau$ for $i > 0$. They are positive given (4), and equal to the half of those under marginal cost pricing.

Furthermore, we must show that (3) is indeed valid. First, no firm is being undercut by its neighbors. This requires $|v_{0t} - p^c_{0t} - v_{it} + p^c_{it}| < \tau$, for $i > 0$. Further, (11) implies that $|v_{0t} - c_{0t} - v_{it} + c_{it} - \tau| < 2\tau$, which is guaranteed by (5). Secondly, the “indifferent” consumers between firm 0 and any firm i must have positive surplus, i.e., $v_{0t} - p^c_{0t} - \theta\tau > 0$, where $\theta = (v_{0t} - p^c_{0t} - v_{it} - p^c_{it})/2\tau$. This is guaranteed by (5).

In model III, (12) satisfies the first-order condition to maximize (8):

$$\frac{2p_{2t} - 2p_{1t} + c_{1t} - c_{2t}}{q_{2t} - q_{1t}} - \frac{2p_{1t} - c_{1t}}{q_{1t}} = 0, \qquad 1 - \frac{2p_{nt} - 2p_{n-1t} + c_{n-1t} - c_{nt}}{q_{nt} - q_{n-1t}} = 0$$

$$\frac{2p_{i+1t} - 2p_{it} + c_{it} - c_{i+1t}}{q_{i+1t} - q_{it}} - \frac{2p_{it} - 2p_{i-1t} - c_{it} + c_{i-1t}}{q_{it} - q_{i-1t}} = 0 \qquad \text{for } i \neq 1, n$$

The second-order derivatives are: $\partial^2\pi/\partial p_1^2 = -2q_{2t}/q_{1t}(q_{2t}-q_{1t})$, $\partial^2\pi/\partial p_n^2 = -2/(q_{nt}-q_{n-1t})$, $\partial^2\pi/\partial p_i^2 = -2/(q_{i+1t}-q_{it}) - 2/(q_{it}-q_{i-1t})$ for $i \neq 1, n$, $\partial^2\pi/\partial p_i\partial p_{i+1} = 2/(q_{i+1t}-q_{it})$, $\partial^2\pi/\partial p_i\partial p_j = 0$ for all $|i - j| > 1$. Similar to model II, the sum of the first row of $\partial^2\pi/\partial \mathbf{p}^2$ is negative, and the rest is zero. Then, this matrix has a quasi-dominant diagonal and is negative definite (McKenzie 1960, Theorem 2). So the first-order condition guarantees the joint-profit maximization.

We need to show that (7) is valid. Substituting (12), we get firms' outputs:

$$x_{1t}^c = 0.5\left(\frac{c_{2t} - c_{1t}}{q_{2t} - q_{1t}} - \frac{c_{1t}}{q_{1t}}\right) \qquad x_{nt}^c = 0.5\left(1 - \frac{c_{nt} - c_{n-1t}}{q_{nt} - q_{n-1t}}\right)$$

$$x_{it}^c = 0.5\left(\frac{c_{i+1t} - c_{it}}{q_{i+1t} - q_{it}} - \frac{c_{it} - c_{i-1t}}{q_{it} - q_{i-1t}}\right) \qquad \text{for } i \neq 1, n$$

As $c''(q) > 0$, $c(q)$ is convex. Given any $\lambda \in (0,1)$, $c[\lambda q_{i-1,t} + (1-\lambda)q_{i+1,t}] < \lambda c(q_{i-1,t}) + (1-\lambda)c(q_{i+1,t})$. Let $\lambda = (q_{i+1,t}-q_{it})/(q_{i+1,t}-q_{i-1,t})$, we get $(q_{it} - q_{i-1,t})c_{i+1,t} + (q_{i+1,t} - q_{it})c_{i-1,t} > (q_{i+1,t} - q_{i-1,t})c_{it}$. This implies $x_{it}^c > 0$ for all $i \neq 1, n$. Further, $x_{1t} > 0$ if $q_{1t}c_{2t} > q_{2t}c_{1t}$. This holds if $c(q)/q$ increases in q. Then, it suffices to show $qc'(q) - c(q)$ is positive. Since $c(0) = 0$ and $c''(q) > 0$, this function increases in q and equals zero when $q = 0$. Hence, it must be positive for any $q > 0$. Moreover, $q_{nt} - c_{nt} > q_{n-1t} - c_{n-1t}$ ensures $x_{nt} > 0$. Thus, all quantities are positive.

Finally, the buyers with the lowest θ's for each product must obtain positive surplus. For firm 1, $\theta = q_{1t}/p_{1t}$, the surplus is positive if $(q_{1t})^2/p_{1t} - p_{1t} > 0$, i.e., $q_{1t} > p^c_{1t}$. This is guaranteed by $q_{1t} > c_{1t}$. For $i > 1$, $\theta = (p_{it} - p_{i-1t})/(q_{it} - q_{i-1t})$. We need $(p_{it} - p_{i-1t})q_{it} > p_{it}(q_{it} - q_{i-1t})$. Under (12), this holds if c(q)/q rises in q, which we just showed above.

Appendix B:

In model I, we define a function L_t as $SW_t - 3CS_t = (2\mathbf{p}_t - \mathbf{a}_t - \mathbf{c}_t)B_t^{-1}(\mathbf{a}_t - \mathbf{p}_t)$. It is easy to verify that, $L_t(\mathbf{p}_t^c) = 0$, $\partial L_t(\mathbf{p}_t^c)/\partial \mathbf{p} = \mathbf{x}(\mathbf{c}_t)$, and $\partial^2 L/\partial \mathbf{p}^2 = -4B_t^{-1}$. The second-order Taylor expansion of L_t around $\mathbf{p}_t^c$ is $\mathbf{x}(\mathbf{c}_t)'\Delta\mathbf{p}_t - 2\Delta\mathbf{p}_t' B_t^{-1}\Delta\mathbf{p}_t$. Since $\mathbf{x}(\mathbf{c}_t) > \mathbf{0}$, B_t^{-1} is positive definite, Hence, $L_t < 0$ for any $\Delta\mathbf{p}_t < \mathbf{0}$. We rule out any deviation with prices higher than $\mathbf{p}_t^c$, because this should never occur.

In model II, we define L_t as $SW_t - 3CS_t - n\tau = \mathbf{x}_t'(\mathbf{p}_t - \mathbf{c}_t) - \mathbf{x}_t'(\partial\mathbf{x}/\partial\mathbf{p})^{-1}\mathbf{x}_t$. In model III, $L_t = SW_t - 3CS_t = \mathbf{x}_t'(\mathbf{p}_t - \mathbf{c}_t) - \mathbf{x}_t'(\partial\mathbf{x}/\partial\mathbf{p})^{-1}\mathbf{x}_t$. Similarly to model I, we have $L_t(\mathbf{p}_t^c) = 0$, $\partial L_t(\mathbf{p}_t^c)/\partial\mathbf{p} = \mathbf{x}(\mathbf{c}_t)$, and $\partial^2 L/\partial\mathbf{p}^2 = 4\partial\mathbf{x}/\partial\mathbf{p}$. In the two cases, we also have $L_t = \mathbf{x}(\mathbf{c}_t)'\Delta\mathbf{p}_t + 2\Delta\mathbf{p}_t'(\partial\mathbf{x}/\partial\mathbf{p})\Delta\mathbf{p}_t$ for any deviation of $\Delta\mathbf{p}_t$. Hence $L_t < 0$ if $\Delta\mathbf{p}_t < \mathbf{0}$.

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